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Frequency of patterns on the plantar configurational areas of the distal sole among Telis of Chhattisgarh.

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Ms: Received 7.12.89; Revised 12.9.91.

Abstract Sole prints of 180 Telis (93 males and 87 females) were analysed for the presence of patterns on the Hallucal distal hypothenar and II, III, IV interdigital areas to indicate the bimanual, bisexual and interpopulation differences. The most common configuration in the hallucal area is the loop in both the sexes of Telis of Chhattisgarh. The second and fourth interdigital areas are usually devoid of patterns. The highest incidence of patterns is recorded in the third interdigital area. Patterns occur more frequently on the distal hypothenar area than in the proximal hypothenar area.

Key Words : Plantar dermatoglyphics, Telis, Chhattisgarh,

1.Introduction

Traditionally, dermatoglyphics has been regarded by anthropologists as an useful tool in the analysis of the relationships among human populations.Though a mumber of reports are available on finger and palmar epidermal ridge configurations the data pertaining to plantar configurations are still scanty Except for a few papers published by Cummins (1930, 1931, 1935), Wilder (1904) Cummins and Midlo (1926, 1927, 1961) and Takeya (1934) in the past and Wichmann (1956), Geipel (1958), Datta (1958, 1966) and Tiwari(1971) during the last few years, no serious attempt has been made to find out the variations in sole patterns among different ethnic stocks.

The present study deals with the frequency distribution of various types of configurations obeserved on the plantar areas of Telis of Chhattisgarh region of Madhya Pradesh. Telis are the occupational caste of oil pressers and oil sellers. But in Chhattisgarh nearly all the Telis are cultivators.

2.Material and methods

Data for the present study consists of 180 prints, including 93 males and 87 females. Every possible care was taken to avoid related male and female subjects in the present study.

The method used in formulating the plantar configurational areas was that proposed by Cummins and Midlo (1961).

3.Results and discussions

The frequencics of the positions of termination of the fibular and distal radiants of triradius 'P'for both the sexes are listed in Table-1. This triradius is present in 39.78% in right soles and 33.34% in left soles of males and 51.73% in right soles and 49% in left soles of females.

Table 1: Frequencies of Fibular and distal radiant of triradius 'p'.

		Fibular radiant						Distal radiant		
	No.	9	8	7	6	5	1"	13	12	11
Males:										
Right	37	2.70	-	89.19	-	8.11	-	62.16	-	37.84
Left	31	3.23	-	87.09	-	9.68	-	32.26	-	67.74
Females:										
Right	45	17.77	-	75.55	-	6.66	-	80.00	-	20.00
Left	36	5.55	-	77.77	-	16.66	8.33	41.66	8.33	41.66

The fibular radiant(Table 1) is directed into the fourth interdigital interval in 88.14% of Teli males. The next common termination is in the distal portion of the fibular border (8.89%) and the third interdigital interval (2.97%). The bipedal differences for the fibular radiant are statistically not significant in Teli males (Chi-square= 0.1032, d.f. = 2, .95>p>.90).

In females the fibular radiant shows a lesser tendency to terminate in the fourth interdigital area (76.66%). Unlike males the next common termination is in the third interdigital interval (5.74%) and termination towards the distal portion of the fibular border (5.17%), comes next to it. The bipedal differences are statistically not significant (Chi-square = 4.2353,d.f. = 2, .20>p>.10.

The termination of fibular radiant in the fourth interdigital area is appreciably more frequent in Teli males than in females. The sex differences are statistically not significant (Chi square = 2.4367,d.f. = 2, .30>p>.20) with regard to terminations of fibular radiant.

The distal radiant most commonly terminates in the second interdigital interval in Teli males (52.79%) and it is followed by termination in the first interdigital interval (47.21%). The bipedal differences for the termination of distal radiants is statistically significant (Chi square = 6.0289, d.f. = 2, .05>p>.02).

In females the frequency of termination in the first interdigital area is 60.83% and in the second interdigital area is 30.83%. The radiant fused with the digital triradius 'a' in 8.33% of females. The bipedal differences for the termination of distal radiants in females were statistically significant (Chi-square = 15.3531,d.f. = 3,.01>p>.001). The sex differences in the terminations of distal radiant are statistically not significant (Chi- square= 5.4426,d.f. = 3,.20>p>.10).

Percentages of pattern types on the plantar hallucal area in both the sexes of Telis are listed in Table 2.

Table 2: Frequency of pattern types on the plantar hallucal area of Telis.

Configurational types	Males (93)			Females (87)		
	Right	Left	Total	Right	Left	Total
W(sp)	7.53	6.45	6.99	1.15	1.15	1.15
W(conc.)	6.45	3.22	4.83	18.39	11.49	14.94
$W^{(sm)}$	10.75	13.98	12.36	6.89	17.24	12.06
C.P.L.	1.08	-	0.54	1.15	6.89	4.02
L.P.L.	-	1.08	0.54	-	-	-
Total	25.81	24.73	25.26	27.58	36.77	32.17
Loops)L^d	67.74	65.59	66.66	65.52	51.72	58.62
L^t	4.30	4.30	4.30	3.54	5.75	4.60
L^f	-	-	-	-	-	-
L^d/L^f	-	-	-	-	-	-
Total	72.04	69.89	70.96	69.06	57.47	63.22
O.F.	2.15	5.38	3.76	3.45	5.75	4.60

The most common configuration in the hallucal area is a loop in both males and females. However, loops are more frequent in males (70.20%) than in females (63.22%), whorls are common in left soles of Teli females (36.77%) and right soles of Teli males (25.81%). The differences between males and females in the occurrence of patterns in the hallucal area are statistically non- significant (Chi-square = 1.226, d.f.- 2,.70>p>.50). The bipedal differences in the occurrence of patterns on the plantar hallucal area of Telis are statistically not significant in both males (Chi-square- 2.0116, d.f.=2,.50>p>.30) and females (Chi-square- 2.5518,d.f.- 2,.30>p>.20).

Mean indices of the pattern intensity in the two sexes of Telis are given in table 3. The value of pattern intensity index is slightly higher in fimales (12.76) than in males (12.14).

Table 3: Mean index of pattern intensity on the plantar hallucal area of the Telis.

	Pattern intensity index		
	Right	Left	Total
Male	12.36	11.93	12.14
Female	12.42	13.10	12.76

The frequencies of types of configurations observed in 2nd, 3rd, and 4th plantar interdigital areas are listed in Table 4.

Table 4: Frequencies of types of configurations in the 2nd, 3rd and 4th interdigital areas of Telis.

	Interdigital II				Interdigital III				Interdigital IV		
	O	L^d	L^p	W	O	L^d	L^p	W	O	L^d	L^p
Males:											
Right	82.79	2.15	12.90	2.15	64.52	24.73	6.45	4.30	92.47	7.53	-
Left	87.09	2.15	10.75	-	72.04	18.28	5.38	4.30	93.55	6.45	-
Females:											
Right	74.71	4.59	19.54	1.14	54.02	40.23	1.15	4.59	89.65	10.34	-
Left	73.56	3.45	22.99	-	58.62	32.18	4.59	4.59	91.95	8.05	-

The frequency of loops in the 2nd interdigital area in males is 13.97% and females 25.28%. The sex differences for the occurrence of pattern in the second interdigital area is statistically not significant (Chi-square 3.7489, d.f.=3,.30>p>.20). The frequency of loops show further decline in the fourth interdigital area, the frequency in males being 6.99% and Females 9.19%. The sex differences are statistically non significant (Chi-square= 3.7489,d.f.= 3,.3>p>.20).

The frequency of loops is the third interdigital area in males is 27.42%, and 39.07% in females. The sex differences in the occurence of patterns in the third interdigital area are statistically non- significant (Chi-square = 5.3684,d.f. = 3,.20>p>.10).

Statistically no significant bipedal differences are observed in the occurence of patterns in the interdigital areas.

The configurations of the distal hypothenar area are listed in Table-5. Thepatterns are common in the distal hypothenar area. The bipedal differences is statistically non-significant in both males (Chi-square=5.0384,d.f. = 2, .20>p>.10 and in females (Chi- squre = 0.1980,d.f.=2,.95>p>.90) sex differences are statisfically significant in the occurrence of configurations in the distal hypothenar area (Chi-square = 7.3678,d.f.=2,.05>p>.20).

Table 5: Frequencies of types of configurations in the distal and proxima hypothenar areas.

	Distal hypothenar			Proximai hypothenar		
	O	V	L	O	L	V
Males:						
Right	16.13	31.18	52.69	89.25	7.53	3.23
Left	6.45	29.03	64.52	87.09	12.90	-
Females:						
Right	27.58	19.54	52.87	82.76	11.49	5.75
Left	25.29	21.84	52.87	89.66	9.19	1.15

Studies with regard to plantar configurations are inadequate for comparasion, yet, an attempt has been made to conclude some important conclusions of various authors. Table -6 presents the frequency distribution of pattern types on distal plantar region of various populations so far studied. Studies conducted on European populations (Wilder,1992, Cummins and Midio, 1926 and Wichmann, 1956) show lesser frequency of whorls on the halucal area as compared to African populations.The frequency of patterns on the hallucal area is low in most of the Mongoloid populations thus revealing comparatively low pattern intensity index. Datta reported few Indian populations of Punjab showing greater affinities to Europoid than to Negroid & Mongoloid. The findings ofTelis are in conformity with the the findings of Cummins (1935). Takeya (1934) and Datta (1966).

The most common configuration in the hallucal area is the loop in both the sexes of Telis of Chattisgarh. The second and fourth interdigital areas are usually devoid of patterns. The highest incidence of patterns is recorded in the htird interdigital area. Patterns occur more frequently on the distal hypothenar area than in the proximal hypothenar area.

Table 6: Frequency distribution of pattern types on distal plantar region of various populations Plantar configurational areas.

POPUL ATION	No.	Hallucal			Inter-II			Inter-III			Inter-IV			Hypothenar distal			Investigation
		W	L	O	W	L	O	W	L	O	W	L	O	L	V	O	
Germans	500	28.1	66.9	5.0	2.8	25.2	72.0	9.5	58.2	32.3	0.1	15.7	84.2	45.9	-	53.9	Winchmann, (1956)
European Americans (M)	200	30.2	66.5	3.2	1.2	27.7	71.0	9.5	62.2	25.2	-	16.2	83.7	-	-	-	Cummins & Midlo, (1926)
European Americans (F)	200	32.7	63.7	3.5	2.7	23.0	72.2	11.2	53.2	35.5	0.5	18.0	81.5	-	-	-	Wilder, (1904)
Jews	100	44.0	49.5	6.5	5.0	41.5	53.5	31.5	56.5	12.0	2.0	19.5	78.5	-	-	-	Cummins & Midlo,(1927)
Negroes of West Africa	100	53.3	39.5	5.0	0.5	32.4	66.4	18.0	53.1	28.7	-	34.8	65.0	-	-	-	Cummins (1930)
African Nigroes	82	68.3	27.4	4.3	2.4	53.6	43.9	32.3	47.5	20.2	-	30.5	69.5	59.8	4.9	35.3	Geipel, (1958)
People of Madagaskar	115	54.8	42.4	3.0	1.3	30.5	68.2	13.0	52.1	34.8	3.0	24.4	72.6	31.8	1.9	66.5	Geipel, (1958)
Indians of S. Maxico & C.America	37	19.1	79.3	1.3	1.4	22.5	76.0	5.7	72.8	21.4	-	15.6	84.2	-	-	-	Cummins (1931)

continued......

Eskimos (St.Lawrence) & King Is)	64	17.3	59.2	22.8	0.8	10.9	88.3	2.3	64.1	33.6	-	9.4	90.6	-	-	-	Cummins & Midlo (1932)
Eskimos (Pt. Barraw)	55	8.1	64.5	27.4	1.6	25.8	72.6	3.2	58.0	38.7	-	3.2	96.8	-	-	-	Cummins (1935)
Chinese	1000	30.6	55.7	12.2	0.1	8.6	91.2	0.6	57.8	41.5	0.05	8.05	91.9	35.9	37.1	27.0	Takeya (1934)
Tibetans (M)	108	18.7	66.1	15.2	-	6.8	93.1	-	51.9	48.1	-	5.1	94.9	45.8	5.6	48.6	Tiwari, (1971)
Tibetans (F)	104	15.8	76.3	7.9	-	6.2	93.2	0.5	49.5	50.0	-	1.9	98.1	39.4	5.3	55.3	Tiwari (1971)
Punjabi Khatri (M)	100	27.0	62.0	11.0	3.0	31.0	66.0	16.5	52.0	31.5	-	23.5	76.5	40.5	3.0	56.5	Datta (1958)
Punjabe Khatri (F)	100	14.0	74.0	11.5	3.0	31.0	65.5	8.5	44.0	47.5	-	9.0	91.0	49.5	9.5	41.0	Datta (1958)
Punjabi Brah.(M)	115	32.3	61.2	6.5	1.8	29.1	69.1	12.5	49.1	38.4	0.4	20.0	79.2	60.0	5.2	34.8	Datta (1966)
Punjabi Brah.(F)	110	22.3	70.4	7.3	4.0	19.0	77.0	5.4	45.0	49.6	-	5.4	94.6	57.3	6.3	36.4	Datta (1966)

continued.....

Khatri (F)																	
Rajput (M)		26.47	67.65	5.88	2.94	35.28	61.78	14.7	35.29	30.0	-	29.41	70.59	2.94	47.06	50.0	Khatriya & Kapoor (1985)
Rajput(F)		18.75	78.12	3.12	6.25	15.63	78.12	-	46.87	53.19	-	21.87	78.13	-	63.33	36.67	Khatriya & Kapoor (1985)
Brahmins (M)		23.33	76.67	-	3.33	40.00	56.67	10.00	46.67	43.33	-	15.00	85.00	-	59.32	40.68	Tiwari & Kapoor (1980)
Brahmins (F)		17.86	67.86	14.28	10.71	26.78	62.50	1.78	51.78	46.43	-	5.36	94.64	-	54.90	45.10	-do-
Rang																	
Bhotia (M)	63	19.40	68.25	12.70	-	14.51	85.49	1.92	49.20	48.88	-	8.06	91.14	-	60.49	48.51	-do-
Johari (M)	64	16.39	72.65	9.37	1.56	13.28	85.16	2.35	54.62	43.03	-	10.15	89.85	-	57.03	46.97	-do-
Johari (F)	68	23.53	69.85	6.67	0.71	2.14	97.15	-	40.81	59.19	-	5.00	95.00	-	49.20	50.72	-do-
Jaads (M)	41	13.41	76.83	14.64	-	4.88	95.12	-	58.54	41.46	-	13.42	86.68	-	63.37	36.63	-do-
Jaads (F)	29	12.06	81.03	3.45	-	8.65	91.35	-	55.33	44.60	-	6.90	93.10	17.2	48.28	50.00	-do-
Teli (M)	93	25.26	70.96	3.76	1.07	13.97	84.90	4.3	27.42	68.2	-	6.99	93.01	58.60	30.10	11.29	Present study
Teli (F)	87	32.17	63.22	4.60	0.57	25.28	74.10	4.59	39.07	56.3	-	9.19	90.80	52.87	20.69	26.43	Present Study

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Palmar flexion creases among Kalar of Madhya Pradesh

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Ms.: Received 28.5.90; Revised 15.9.90.

Abstract. The present paper deals with the study of palmar creases in their bimanual, bisexual and ethnic distribution among kalar of Madhya Pradesh. Kalar is an occupational caste of distillers and sellers of fermented liquor and is a genetically diverse group of Indian Population.

1. Introduction

Palmar flexion creases have always been considered as the domain of the palmist but in the present times the utility of palmar creases has been fully recoginzed.The work available in this direction clearly indicates that this trait can easily find its place in the field of physical Anthropology, medical Science and human Genetics . To establish the importance of palmar crease in population studies, the work of Portius (1937), Wurth (1937), Fischer (1949), Buchi (1954), Lestrange (1969), Kimura (1968), Gyenis & Gyorgy (1971), Bali and Chaube (1971) & Chaube (1977, 79 86, 90) are worthwhile to mention.

It is the only Simian Crease which finds frequent references in this context. Most of the existing methods of crease analysis are Simian Crease oriented and subsequent influence of this limitation appears to have hampered the development of new methods, which could be used to accommodate all creases in its fold.

In view of these limitations an attempt was made by Bali and Chaube (1971) to classify major creases of the palm on the basis of thier origin.

The present study deals with the study of palmar creases in thier bimanual, bisexual and ethnic distribution among kalar of Madhya Pradesh, kalar is a occupational caste of distillers and sellers of fermented liquor, and has not been studied so far as for as the palmar creases studies is concerned. This group is a genetically diverse group of Indian population.

2 . Material and methods

Bilateral palm prints from both sexes of Kalars were collected from different places of M.P. To obtain good prints the method suggested by Cummins and Midlo (1961) has been followed. The sample includes 450 males and 450 females. The analysis of the data have been done according to the method described by Bali and Chaube (1971). In this method three major creases, distal transverse crease, proximat transverse crease and Raidial longitudinal crease has been classified on the basis of their common point of origin. If these creases originate from the single base point on the radial side of the palm, they are called

single radial base crease (SRBC), if they originate from couble base point, they are called double radial base creases (DRBC) and when originates from triple base points, they are called triple radial base creases (TRBC).

These main type of creases are further classified into their subtypes on the basis of their initial split or bifurcation of transverse crease. The subtypes of SRBC are S_1, S_2 S_3, S_4, S_5 and the subtypes of DRBC are D_1, D_2, D_3, D_4, D_5, & D_6. The subtypes of TRBC has not been reported so far (Fig- 1 to 4).

3. Results and discussions

Table 1 presents the bimanual and bisexual distribution of three main types of palmar creases. It could be seen from the table that kalar males exhibits highest frequency of DRBC (72.33%) followed by the frequency of SRBC (18.55%) and TRBC (9.11%). In Kalar females,again highest frequency is shown by DRBC type (75.8%) while the next higher frequency in the descending order is shown by TRBC (12.33%) and the lowest frequency is shown by SRBC type (11.77%). In the frequencies of SRBC type both males and females show sinistral dominence (Males : Rt-15.55%, Lt-21.55%; Females: Rt- 10.88%, Lt-12.66%.) Considering the DRBC Types the males exhibits dixtral dominece (Rt-73.77%, Lt-70.88%) and the females shows sinistral dominence (Rt-75.77%, Lt-76.00%). The frequency of TRBC shows dextral dominence in both sexes (Males : Tt-10.66%, Lt- 07.55%; Females : Rt-13.33%, Lt-11.33%).

Table-1: Distribution of Main types of palmar flexion creases (Transverse) Among Kalar males & Females of Madhya Pradesh

S.No.	Main types of Palmar Creases		Males			Females		
			Rt	Lt	Rt + Lt	Rt	Lt	Rt+Lt
1.	SRBC		70	97	167	49	57	106
			(15.55)	(21.55)	(18.55)	(10.88)	(12.66)	(11.74)
2.	DRBC	Ab	332	319	651	341	342	683
		%	(73.77)	(70.88)	(72.33)	(75.77)	(76.00)	(75.88)
3.	TRBC	Ab	48	34	82	60	51	111
		%	(10.66)	(07.55)	(09.11)	(13.33)	(11.33)	(12.33)
	Total	Ab	450	450	900	450	450	900
		%	99.98	99.98	99.98	99.98	99.99	99.95

Chi square value for bilateral difference in Males - $\chi^2 = 7.00$*

Chi Square value for bilateral difference in females - $\chi^2 = 1.32$

Chi square value for bisexual difference (Hands combined) - $\chi^2 = 18.75$*

* Significant upto 5% level or below. Rt - Right hand, Lt - Left hand, Ab - Absolute frequency.

χ^2 values suggests significant bilateral difference in case of males while it is not significant in case of females. A significant bisexual variation is exhibited by the trait.

Table 2 represents the percentile frequencies of the subtypes of palmar creases among Kalar males and females. It is seen that among the subtypes of SRBC the males exhibit higher frequency of 54 (11.11%) and the lowest of S_1 (0.77%). Among females higher frequency of S_5 (4.55% and the lowest of S_2 is observed (0.77%). A striking bisexual and bilateral difference is visible in the case of subtypes of SRBC. Among the subtypes of DRBC it can be inferred that Kalar males exhibit higher frequency of D_5 ((Rt-32.33%, lt- 29.11%) followed by the frequency of D_4 (Rt- 21.55%, Lt-24.88%), while the Kalar females exhibit higher frequency of D_4 (Rt-36.22%, Lt-34.88%) and followed by the frequency of D5 (Rt 20.00%, Lt 30.00%). A striking bisexual difference is present in the subtypes of DRBC. Subtype T_1 exhibit the similar trend in its distributiion as exhibited by TRBC. A marked bisexual diffirence has been exhibited by the frequency of T_1 (males : 9.11% and females : 12.33%).

Due to the scarcity of data on palmar flexion creases from Indian population the kalars of Madhya Pradesh have been compared with them only to assess population variation for the main type of palmar creases.

The distribution of the SRBC, DRBC and TRBC types of palmar creases in some of the Indian population is shown in table 3. It has been observed that in all the population groups the frequency of DRBC is higher as compared to SRBC and TRBC. The table exhibits that in males the frequency of DRBC ranges from 64.52% in Onge of Andaman to 85.78% in Punjabi brahmins of Delhi and in females it ranges between 41.66% in onge of Andaman to 77.00% in Nairs of A.P. and Maharastrian Thakurs of M.P. In males, the lowest frequency 6.84% of SRBC is observed in Punjabi's of Delhi and highest 22.58% in Onge of Andman, while in females the lowest frequency 4.00% is observed in Maharastrian Thakur and the highest 23.07% in Brahmins of M.P. In males the frequency of TRBC varies from 4.54% in yadav of M.P. to 14.00 per cent in Maharastrian Brahmins of M.P. In females, the frequency of TRBC ranges between 11.00% in Nairs of A.P. and 39.58% in Onge of Andman. The frequencies of SRBC, DRBC & TRBC of the present study falls within the range of variations observed in the other population groups. But more data are needed for comparison with other population groups of India to enunciate a specific trend of palmar creases.

Table-2: Distribution of the Subtypes of palmar flexion creases (Transverse) Among the Kalars of Madhya Pradesh

S.No.	Sub types of Palmar Creases		Males Rt	Males Lt	Males Rt + Lt	Females Rt	Females Lt	Females Rt+Lt
1.	S_1	Ab	1	6	7	11	6	17
		%	(00.22)	(01.33)	(00.77)	(02.44)	(01.13)	(01.88)
2.	S_2	Ab	9	12	21	3	4	7
		%	(02.00)	(02.66)	(02.33)	(00.66)	(00.88)	(00.77)
3.	S_3	Ab	6	12	18	5	3	8
		%	(01.33)	(02.66)	(02.00)	(01.11)	(00.66)	(00.88)
4.	S_4	Ab	46	54	100	12	21	33
		%	(10.22)	(12.00)	(11.11)	(02.60)	(04.66)	(03.66)
5.	S_5	Ab	8	13	21	18	23	41
		%	(01.77)	(02.88)	(02.33)	(04.00)	(05 11)	(04.55)
6.	D_1	Ab	25	19	44	5	6	11
		%	(05.55)	(04.22)	(04.88)	(01 11)	(01 33)	(01.22)
7.	D_2	Ab	14	3	17	5	7	12
		%	(03.11)	(00.66)	(01.88)	(01.11)	(01.55)	(01.33)
8.	D_3	Ab	5	4	9	10	13	23
		%	(01.11)	(00.88)	(01.00)	(02.22)	(02.88)	(02.55)
9.	D_4	Ab	97	112	209	163	157	320
		%	(21.55)	(24.88)	(23.22)	(36.22)	(34.88)	(35.55)
10.	D_5	Ab	145	131	276	126	135	261
		%	(32.22)	(29.11)	(30.66)	(28.00)	(30.00)	(29.00)
11.	D_6	Ab	46	50	96	32	24	56
		%	(10.22)	(11.11)	(10.66)	(07.11)	(05.33)	(06.22)
12.	T_1	Ab	48	34	82	60	51	111
		%	(10.66)	(07.55)	(09.11)	(13.33)	(11.33)	(12.33)

Table-3: Frequency of the Main types of Palmar Flexion Creases Among some of the Indian Population Groups

S.No.	Populations	Sex	No.	% Frequency of			Investigater	Year
				SRBC	DRBC	TRBC		
1.	Brahmins (M.P.)	M	130	11.15	81.53	7.31	Chaube, R.	(1977)
2.	Brahmins (M.P.)	F	130	23.07	63.85	13.08	-do-	
3.	Bhangi (M.P.)	M	75	16.67	77.33	6.00	-do-	
4.	Bhangi (M.P.)	F	75	16.66	72.00	11.33	-do-	
5.	Punjabis (Delhi)	M	95	6.84	85.78	7.37	-do-	
6.	Punjabis (Delhi)	F	95	5.27	64.21	30.52	-do-	
7.	Nair (A.P.)	M	300	8.33	81.66	10.00	Chaube, R.	(1979)
8.	Nair (A.P.)	F	100	19.00	77.00	11.00	-do-	
9.	Yadav (M.P.)	M	220	12.27	83.18	4.54	-do-	
10.	Maharastrian Brahmins (M.P.)	M	100	12.00	74.00	14.00	Chaube, R.& S. Rai	(1986)
11.	Maharastrian Brahmins (M.P.)	F	100	14.00	70.00	16.00	-do-	
12.	Maharastrian Thakur (Maratha)	M	90	13.33	75.56	11.11	-do-	
13.	Maharastrian Thakur (Maratha)	F	100	4.00	77.00	19.00	-do-	
14.	Nicobarese	M	59	17.79	68.64	13.56	Chaube, R. et.al.	(1990)
15.	Nicobarese	F	54	8.33	67.59	24.07	-do-	
16.	Onge (Andaman)	M	31	22.58	64.52	12.90	-do-	
17.	Onge (Andaman)	F	24	18.75	41.66	39.58	-do-	
18.	Kalar (M.P.)	M	450	18.54	72.30	9.11	Present Study	
19.	Kalar (M.P.)	F	450	11.74	75.87	12.33	-do-	

4. Conclusion

In the present study the sexual dimorphism shows sihnificant variations in the frequencies of SRBC, DRBC and TRBC types and their subtypes also. Significant bilateral differences are shown by the Kalar males but in case of females it is not significant. When Kalar males and females were compared with the other population groups of India, it was found that they show resemblance with the other population groups. Hence it can be deduced that both sexes of Kalars have been derived from the same population and the few differences between Kalar and the other population groups may be due to either strict endogamy or because of genetic drift. But a detailed and extensive study of palmar creases would be needed to substantiate the aforesaid hypothesis.

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Incidence of colour blindness among three groups of Binjhwars of Chhattisgarh

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Abstract. Binjhwar is a Dravidian tribe and is considered to be an offshoot of the Baiga of Mandla and Balaghat district. In the present study the incidence of colour blindnees among three engogamous groups of Binjhwars have been recorded. The present investigation was carried out on 240 Binjhwars of Bade Binjhwars (130 males and 110 females), Saunjharas (90males and 100 females) and 210 in Binjhias (110 males and 100 females) with the help of Ishihara Charts. The incidence of colour blindness were absent in all the three groups in females whereas in males, the hightest frequency was observed among the Bade Binjhwars (5.38%) followed by Saunjhars (2.22%). On the whole, low incidence of colour blindness was observed.

The three groups of Binjhwars show a non-sighificant difference with respect to the incidence of colour vision defect. The present populations were also compared with other tribal groups of Madhya Pradesh (réported by different investigators). All the three sub-groups of Binjhwars did not show significant difference with any other tribal groups of Madhya Pradesh except with the Bhatra, Muria, Dhurwa and Gond.

Key Words :- Colour blindness : Binjhwar ; Chhattisgarh.

1. Introduction

Clemant's (1930) work is one of the earliest report available for the population difference in colour blindness. Colour blindness occurs in many Varieties from complete blindness for all colours to blindness for red to blindness for green (Whitting Hill, 1965). The red green type of colour blindness has been proved to be an X- linked recessive trait. This type of colour blindness is further divided ito two main types viz., anomalous trichromatism and dichromatism. So, the red blind (Proton) person may be Protanomalous or Protonopic and similarly, green blind (deutan) person may be deuteranomalous or deuteranopic. Recent studies show that the incidence of colour blindness varying in different ethnic population was attributed to relaxed natural selection (Past 1962, Pickford 1963). It was concluded that the colour- vision defect would be more damaging in primitive people. Young (1801) & Bhattacharya (1965), Showed that the colour blindness may be due to absence of some pigments. However, various investigators (Mering, 1905 ; Adam, 1970, Dutta, 1966 Cruzcake, 1970 ; Katmus, 1972 ; Malhotra, 1978, Tiwari, 1983 & Bansal, 1974) have Shown the variable expressivity of the gene responsible for colour blindness.

The Binjhwar is a little known Dravidian tribe of Madhya Pradesh. According to 1971 Census total number of Binjhwar is 57,152. Out of the 47 tribes of Madhya Pradesh recorded by Naik (1964), the Binjhwars are comparatively civilized Dravidian tribe (Russell and Hiralal, 1939) concentrated in rural area of Mahasamund tahsil of Raipur district and Katghora tahsil of Bilaspur district. The rest are distributed unevenly in the districts of Mandla, Seoni, Balaghat, Sarguja, Raipur and Bastar. According to Russell and Hiralal (1939), there is little or no doubt that Binjhwars are an off shoot of the primitive Baiga tribe of Mandla and Balaghat of M.P.. They occupy the Satpara and Maikal Hills of the north of Chhattisgarh plain. Binjhwars are divided into four groups namely (i) Bade Binjhwar, (ii) Saunjhara, (iii) Binjhia and (iv) Birjhia. Now a days Birjhia sub- group is not found.

The purpose of the present paper is to show the results of the investigation concerning red green colour vision dificiency among the Binjhwars of Chhattisgarh in Madhya Pradesh.

2. Material and methods

A total of 640 individuals from three groups of Binjhwar, both males and females between17 and 59 years were tested for red green colour vision. The samples were drawn from different villages of two districts (Raipur and Bilaspur) of Madhya Pradesh. The present investigation was carried out on 240 Binjhwar of Bade Binjhwar (130 males and 110 females), 190 of Sounjhara (90 males and 100 females) and 210 of Binjhia (110 males & 100 females). Individuals were tested with the help of Ishihara (1960) plates.

The plates were shown to each subject separately in adequate day light. As they were illiterates, they were requested to trace the winding lines between the X's in plates from 26 to 37 with the help of a fine brush.

3. Results and discussions

The results of colour vision defects for three endogamous groups of Binjhwars are given in Table No. 1. The incidence of colour blindness was absent in all the three groups in females whereas in males, low incidence of colour blindness was observed. However, the incidence of colour blindness in males was observed to be highest among the Bade Binjhwar (5.38%) followed by the Saunjhara Binjhwar (2.22%). Insignificant sex difference was observed. The three groups do not differ significantly from the another at 5 percent level of probability.

So far, very few investigations have been conducted on the frequency of colour blindness in tribal populations of Madhya pradesh. Negi and Ahmed (1962, 1963) tested colour blindness on Bhatra, Muria Dhurwa, Northern Dhurwa and Dorla tribes of Bastar, and observed them to be 5.06%, 4.12%, 5.88%. 3.92% and 2.36% colour blind respectively. The frequencies among Gond, Hill Maria and Kanwar have been reported by Negi (1963) to be 5.69%, 1.18% and 3.30% respectively. Benerjee and Dhar (1984) observed 2.33% and 2.88% colour blindness respectively. The highest frequency was found among the Dhurwa and the lowest frequency was found among the Hill Maria.

Table-1: Incidence of colour- blindness among three groups of Binjhwars

Population	Sex	Total number tested	Normal		Colour blind	
			No.	Percent	No.	Precent
Bade Binjhwar	M	130	123	94.62	7	5.38
	F	110	110	100.00	-	-
Saunjhara	M	90	88	97.72	2	2.22
	F	100	100	100.00	-	-
Binjhia	M	110	106	96.33	4	3.67
	F	100	100	100.00	-	-
Pooled	M	330	317	96.06	13	3.94
	F	310	310	100.00	-	-
Total		640	627	97.97	13	2.03

Group Differences	Chi-square Value	Probability	Remarks
BadeBinjhwar and Saunjhara	1.3544	0.30 > p > 0.20	Insignificant
Bade Binjhwar and Binjhia	0.4023	0.70 > p > 0.50	Insignificant
Saunjhara and Binjhia	0.3335	0.70 > p > 0.50	Insignificant

The present study has been compared with other ten tribal population groups of Madhya Pradesh so as to show the variability of this trait (Table-2). The present population do not show any significant difference with the population groups of Madhya Pradesh except with the Bhatra, Muria, Dhurwa and Gonds.

Table-2: Inter population difference with respect to red- green colour vision dificiency in tribal populations of Madhya Pradesh

Population	N	Colour	deficient	Chi square value. d.f-i	Source
All Binjhwars	640	13	(2.03)		Present Study
Bhatra	237	12	(5.06)	5.752*	Negi and Ahmed 1962
Muria	291	12	(4.12)	3.354*	Negi and Ahmed 1962
Dhurwa	170	10	(5.33)	1.2429*	Negi and Ahmed 1962
N. Dhurwa	51	2	(3.92)	0.7160	Negi and Ahmed 1962
Dorla	127	3	(2.36)	0.0403	Negi and Ahmed 1962
Gond	123	7	(5.70)	4.323*	Negi, 1963
Hill Maria	25	1	(1.13)	0.2971	Negi, 1963
Kanwar	91	3	(3.30)	0.5277	Negi, 1963
Bhil	86	2	(2.33)	0.228	Banerjee and Dhar, 1984
Bhilala	104	3	(2.88)	0.3144	Banerjee and Dhar, 1984

Figures in the parentheses indicate percentage.
* Significant at 5% level of probability.

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Somatoscopy of head hair among Jaunsari Rajputs of Dehradun District, Uttar Pradesh - A brief Note

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Abstract. The present study deals with the somatoscopic observation on hair colour and hair form of scalp region among the Jaunsari - Rajput of Dehradun district in Uttar Pradesh. The observation in the present study accounts a higher frequency of dark coloured hair in adult, and dark and brown coloured hair in children than any other types of colour. The hair form of the Jaunsari Rajput when compared with other racial groups indicates its similarity with mongoloid type of hair. A non-singificant association between the sex and form of hair has also been observed.

1. Introduction

Morphological and histological characteristics of human scalp hair has found its importance for racial classification, in forensic investigations, nutritional aspect and other biological studies. Anthropologists for a long time has recognized, the colour, form and texture of the human scalp hair as a criterion for racial classification (Huxley, 1865, Deniker, 1900, Martin, 1928, Hooton, 1946). Studies on ethnic variation of these histomorphological characteristics among India Populations are limited and thus needs to be investigated.

The Khasas Rajputs of Jaunsar Bawar are the tribal people inhabiting in the cis-himalayas located at the north-western corner of the state of Uttar Pradesh and are well known for polyandrous society (Majumdar, 1962). In the present study an attempt has been made to account for the morphological differences of hair colour and hair form of the scalp region among the Rajputs of Jaunsar- Bawar pargana of Dehradun district in Uttar Pradesh and further to find sexual variation, if any, with hair form.

2. Materials and methods

Somatoscopic observations on hair colour and hair form on the scalp hair were made on 157 individuals (Jaunsari-Rajputs) belonging to both the sexes (94 males, 63 females) and of various age groups from Nagthat and its surrounding villages namely Bisoi, Diuna, Kwasa, Laccha and Thana in the Jaunsar-Bawar area of Dehradun district in Uttar Pradesh.

For somatoscopic observation on hair colour, the threefold classification as described by Banerjee (1957) applicable to Indian population variability which is as -

1. Dark colour 2. Brown colour 3. Between 1+2 (i.e. between dark colour and brown colour) was followed whereas for hair form, the classification as given by Mollison (1938) is adopted, which is as - 1. Leiotrichous - (a) Straight (b) Smooth (c) Flat wavy; 2. Cymotrichous - (d) Broad wavy (e) narrow wavy (f) curly

The Chi-square test has been used to compare hair form and sexes. The word 'significant' is used to mean significance probability at 5% level.

3. Results and discussions

As far as the head hair colour is concerned it seems that not much work has been conducted for variation of head hair colour among Indian population. If we agree with the opinion given by Banerjee (1956) then the Fischer scale (1928) is not applicable to head hair colour variation in this country.

Table 1. Percentage occurence of hair colour among male and female in the present study.

Hair Cololur	Male (94)				Female (63)			
	Adult		Children		Adult		Children	
	No.	%	No.	%	No.	%	No.	%
Dark (1)	8	66.66	17	20.73	6	60.00	1	1.89
Brown (2)	2	16.67	24	29.27	3	30.00	13	24.53
Between 1 & 2	2	16.67	41	50.00	1	10.00	39	73.58

In the present study it is evident from Table 1, that among the adults of both sexes the percentage occurrence of dark coloured hairs (66.66% in males, 60.00% in females) are observed to be higher than the percentage occurrence of brown coloured hairs (16.67% in males, 30.00% in females). The hair colour between dark and brown (between 1 and 2) is observed to be in lesser frequency (16.67% in males, 10.00% in females).

Among the children of both the sexes, the hair colour between dark and brown (between 1 and 2) is observed to be more frequent (50% in males, 73.58% in females). The observations in the present study accounts for higher frequency of dark coloured hairs in adult population while the hair colour between dark and brown is observed to be more among childrens.

The percentage occurence of hair form among 157 individuals of both the sexes is given in Table 2. The percentage occurence of straight hair among males is much higher than in females. In respect of smooth flat wavy and broad wavy type of hairs, higher frequency is observed among females than in males. The frequency of narrow wavy is observed to be much higher in males. However, there was not much difference in the percentage occurence of curly hair among both the sexes.

Table 2: Percentage occurence of hair form among Male and Female in the present study

Hair Form	Male (94) No.	Male (94) %	Female (63) No.	Female (63) %
Straight	46	48.94	20	31.75
Smooth	16	17.02	13	20.63
flat wavy	5	5.32	5	7.94
Broad wavy	17	18.09	21	33.33
Narrow wavy	7	7.45	2	3.17
Curly	3	3.19	2	3.18

The hair form of the Jaunsari-Rajputs in the present study is compared with the type of hair form among the three types of primary races (Coon, 1966). Generally speaking the characteristics of hair form in the Mongoloid, Caucasoid and Negroid population are: Mostly straight and partly slight wavy; Ranges from straight to curly but mostly wavy; Kinky and frizzly hair, respectively.

The hair form of the Jaunsari-Rajputs on comparison with above indicates a probative range of Monogoloid type of hair.

Further to account for the association of hair form and sex of the individual, the Chi-square test (2x2 contingency table) is applied as given in Table 3. The calculated value of Chi- square ($\chi2$ = 2.05, at 1 d.f.) gives a non-significant association between hair form and sex of the individual.

Table 3: 2 x 2 Contingency table - variation of hair form and sex

	Male	Female	Total	χ^2 - value
Leiotricy	67	38	105	2.05
Cymotricy	27	25	52	
	94	63	157	

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Physico-chemical characters of a Steel Plant Effluent for seven continuous days.

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Ms: Received 29.3 90; Revised 3.9.90

Abstract. Liquid effluent from Bhilai Steel Plant (B.S.P.) Bhilai were analysed for physico-chemical variables, for seven continuous days, about 100 m upstream to the point of its joining a river. Some of the variables with higher mean values were nitrate-nitrogen (167.4 mg/l) ammonia-nitrogen (90.9 mg/l), sulfate (915.6 mg/l) and iron (364.8 mg/l) while variables like chloride (22.93 mg/l) nitrite nitrogen (4.29 mg/l) and phosphate (0.346 mg/l) exhibited comparatively lower concentrations Ammonia concentration was beyond ISI permissible limit of 50 mg 1

Key words. Steel plant, liquid effluent, physico-chemical characters, permissible limit.

1. Introduction

Bhilai Steel plant (B.S.P.), Bhilai is one of the largest integrated steel mill in India. The combined liquid waste from different units from entire factory area as well as the domestic waste water is discharged through two channels. One channel joins the river Shivnath while the other channel called the Somninala, joins the river Kharoon (Fig.1). To ensure proper treatment of such effluent, ISI has prescribed limits on the permissible level for some effluent constituents (IS: 2490,1963; IS:8073; 1976). Nevertheless, there are no reports about the characteristics of the combined effluent water discharged from the integrated steel mills. The present studies aims to assess the physico-chemical property of effluent water from an integrated steel mill, Bhilai Steel Plant (BSP) Bhilai, just before its release into Kharoon river.

2. Materials and methods

BSP effluent water samples were collected from Somni Nala, at about 100 m upstream to the release point into the Kharoon river, The samples were collected between 9.00 a.m. to 10 a.m., for seven continuous days, from 29th Oct. to 4th Nov. 1988. pH, electrical conductivity and total dissolved solids were determined at the time of sampling with the help of a Century portable water analyser kit, while remaining variables were determined after transporting the sample to the laboratory within 20 minutes of sampling. Nitrate-nitrogen was determined following Jackson (1973) while all other remaining variables were determined following Standard Methods (APHA-AWWA-WPCF, 1975).

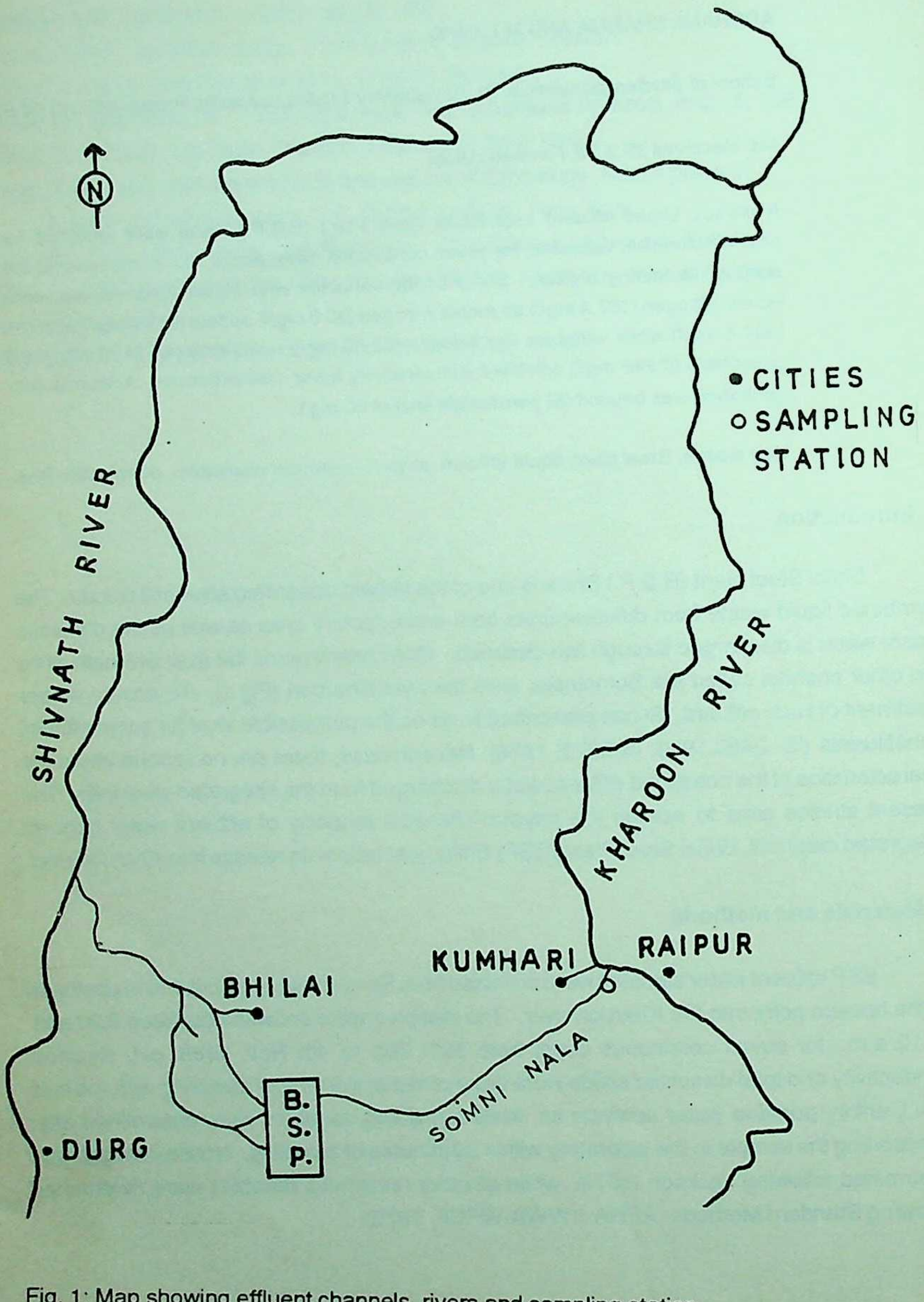
N
● CITIES
○ SAMPLING STATION
SHIVNATH RIVER
KHAROON RIVER
KUMHARI
RAIPUR
BHILAI
B. S. P.
SOMNI NALA
DURG

Fig. 1: Map showing effluent channels, rivers and sampling station.

3. Results and discussions

The values for all the physico-chemical characters are shown in Table 1.

Table 1: Values of physico-chemical characters, for seven continuous days in the year 1988.

	Dates of sampling						
Parameters*	29-10	30.10	31-10	1-11	2-11	3-11	4-11
Temperature °C	25.4	25.2	25.6	26.6	26.0	26.1	25.5
pH	7.17	7.29	7.72	7.42	7.54	7.55	7.42
Electrical conductivity m. mhos/cm	0.382	0.395	0.408	0.485	0.462	0.446	0.459
Total dissolved solids	179	192	198	244	233	222	230
Alkalinity	199.5	139.5	134.9	111.6	130.2	127.9	136.7
Free CO_2	0.79	1.06	0.84	1.23	0.66	0.97	0.44
Total CO_2	106.0	123.8	119.5	99.4	115.2	113.5	120.7
Dissolved oxygen	5.85	7.93	9.78	4.55	5.34	5.25	5.25
Oxygen saturation (%)	68.05	92.17	115.66	54.75	63.12	62.08	62.08
Chloride	22.5	27.0	20.7	27.0	20.7	24.3	18.9
Hardness as Ca	34.63	38.96	83.69	62.04	47.62	60.60	47.62
Hardness as $CaCO_3$	86.4	97.2	208.8	154.8	118.8	151.2	118.8
Total hardness as $CaCO_3$	140.4	133.2	302.4	237.6	187.2	248.4	208.8
Magnesium	13.18	8.78	22.84	20.20	16.69	23.72	21.96
Nitrate- N	236.1	152.9	136.4	167.5	147.4	178.5	152.9
Nitrite- N	142.19	38.80	263.05	71.32	45.70	45.71	29.59
Sulfate	592.7	707.2	769.3	1337.9	1088.7	938.7	974.9
Phosphate	0.75	0.68	0.23	Nil	0.30	0.23	0.23
Silica	0.091	0.095	0.111	0.146	0.099	0.099	0.095
Iron	58.33	89.74	97.70	1688.58	233.70	184.69	201.02

* Except for pH, parameters with which no units are mentioned are in mg/l.

Temperature exhibited a very little variation within seven sampling days, with a range within only 25.2°C to 26.6°C and a mean of 25.8°C. pH was also similar to temperature in having a very narrow range of variation within 7.17 to 7.72 with a mean of 7.44. Electrical conductivity varied between 0.382 m. mhos/cm to 0.485 m.mhos/cm with a mean of 0.434 m.mhos/cm. The amount of total dissolved solids exhibited a little more variation ranging from 179 mg/l to 244 mg/l with a mean of 214 mg/l. Phenolphthalein alkalimity was absent throughout the study period and hence the methyl orange alkalinity was the same as the total alkalinity or bicarbonate alkalinity. This also varied within a relatively narrow range of 111.6 mg/l to 139.5 mg/l with a mean of 128.6 mg/l. Amount of free CO_2 remained relatively low throughout the study period with a range of only 0.44 mg/l to 1.23 mg/l and a mean of 0.86 mg/l, but the amount of total CO_2 was comparatively very high, ranging from 99.4 mg/l to 129.8 mg/l with a mean value of 114.0 mg/l. Concentration of dissolved oxygen exhibited more than twofold variation, ranging from 4.55 mg/l to 9.78 mg/l and similarly the saturation of the effluent with oxygen varied between 54.75% to 115.67%. Chloride concentration was found to be maintained at lower level with a range of only 18.3 mg/l to 27.0 mg/l and a mean of 22.9 mg/l. Calcium hardness as calcium ranged from 34.63 mg/l to 83.69 mg/l with a mean of 53.59 mg/l, while calcium hardness as calcium carbonate, exhibited proportionately higher values of about 2.5 times more than the hardness values as calcium, ranging from 86.4 mg/l to 208.8 mg/l with a mean of 133.7 mg/l. Total hardness values ranged from 133.2 mg/l to 302.0 mg/l with a mean of 208.3 mg/l. Magnesium concentration, the other constituent to determine the hardness, had generally lower values with a range of 8.78 mg/l to 23.72 mg/l and had a mean of 18.20 mg/l. Nitrate-nitrogen maintained a very high concentration throughout the seven days study period. The values ranged from 136 4 mg/l to 236.1 mg/l with a mean of 167.4 mg/l. This was the most abundant form of nitrogen amongst the three forms investigated presently. Nitrite-nitrogen had the lowest value amongst the three forms of nitrogen with a range of 0.71 mg/l to 13.91 mg/l and a relatively small mean value of 4.29 mg/l. Ammonia-nitrogen had on the average intermediate concentration between nitrate and nitrite nitrogen but with a wider range of 29.59 mg/l to 263.05 mg/l and a mean value of 90.91 mg/l. Sulfate concentration, the third factor to determine hardness, was maintained high throughout the study period and also with a wide range of variation from 592.7 mg/l to 1337.9 mg/l and a mean of 915.6 mg/l. Phosphate and silica both had lower concentration on all the seven days with a range of zero to 0.75 mg/l and a mean of 0.35 mg/l for phosphate and with a range of 0.091 mg/l to 0.146 mg/l and a mean of 0.105 mg/l for silica. Iron concentration exhibited higher values on all the dates with a minimum of 58.33 mg/l to a maximum of 1688.58 mg/l and with a mean of 364.82 mg/l.

As expected T.D.S. and conductivity (1.00), alkalinity and total CO_2(1.00) calcium hardness as Ca and $CaCO_3$ (1.00) and calcium hardness and total hardness(0.97) had either perfect or almost perfect correlation with each other. Similarly sulfate and conductivity(0.97), sulfate and total dissolved solids(0.96), silica and nitrite(0.96) and silica and iron(0.94), also had very high level of correlation with each other. Of the limits prescribed in IS : 2490.(1963). only ammonia nitrogen was found to be beyond permissible limit in the effluent, before it is released to the Kharoon river.

Acknowledgements

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Seasonal and Photoperiodic control of the activity and daily exodus of a colony of cave-dwelling bat.

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Ms: Received 5.4.90; Revised 17.2.92.

Abstract. The onset and the end of activity in a colony of approximately 1,500 cave dwelling bats were observed during four different months (December 22,1987;March 22,1988; June 21,1988 and September 23,1988). Results reveal that diurnal rhythm in activity patterns of the bat colony is greatly influenced by natural light:dark cycle. Computation of median timings for emergence and returning activity of the colony favours sunset as the strongest synchronizer of the activity rhythm of the colony.

1. Introduction

The timings of sunset and sunrise vary as a function of the season. The organisms that derive information from either sunset or sunrise timing exhibit comparable variations in different aspects of their overt circadian rhythmicity. This has been amply demonstrated in several avian and mammalian species, including bats, of temperate regions (Kunz 1974; Voute et al 1974; Erkert 1978; Swift 1980). However, few reports on the daily onset and end of acitvity of tropical cave dwelling bats are also available (Subbaraj and Chandrashekaran 1977; Usman 1981; Marimuthu 1984).The above studies were conducted at a relatively lower latitude (9^oN). In the present work, the emergence and returning activities were studied in a cave dwelling bat, H.fulvus at a higher latitide (18^oN).

2. Materials and methods

Study area a natural subterranean limestone cave, popularly known as"Kotumsar Cave" lies on the bank of the river Kanger in the Kanger Valley National Park, Jagdalpur Bastar district, (M.P.)(Lat.:$18^o52'$N:Long.: $81^o56'$E)at an altitude of 560 meters. A vertical fissure in the wall of the hill provides the main entrance to the irregular chambers of the cave. It has a narrow and twisted opening measuring about 15 meters in depth. The main tunnel of the cave extends about 200 meters and there are number of downward offshoots. The numerous branchings of the tunnel inside the cave, downwards as well as sidewards, make The interior of the cave look like a honey comb in structure. The interior of the cave has numerous stalactite and stalagmite formations and there are several small pools fed by seepage water .The temperature inside the cave remains constant throughout the year ($28+1^oC$). A colony of about 1,500 bats, Hipposideros fulvus inhabits this cave and several cave pockets are used by the bats as their roosting sites.

The timings of emergence and return ofthe first bat were recorded visually during dusk and dawn, respectively at the mouth of the cave (main entrance) during different seasons winter (December) spring (March), Summer (June) & autumn (September). Observations were made until the last bat emerged or returned. Total number of batsemerged or returned during a single daily activity cycle was noted. However,observations were not made throughout the night.

The median timings of emergence or of return were computed for calculating the phase angle difference in relation to seasonal variations in the timings of sunset and sunrise. The time lag between the timing of emergence of the first bat and the timing of return of the last bat has been considered as the duration of daily activity of the bat colony.

3. Results.

3.1 Emergence activity.

The pre-emergence activity was recorded just prior to the beginning of sunset, during all the four seasons. The length of preemergence activity was found to be different during different months of study. The longest preemergence activity was recorded on June 21, 1988 and the shortest on December 22,1987. The length of preemergence activity was statistically significantly correlated with the length of the day (r=1.0; P<0.01).

In the beginning emergence of bats occurred singly or in small groups.Then the intensity of emergence activity increased rapidly and declined slowly. Median timings of emergence activities were computed and phase angle differences were determined in relation to respective sunset timings during different months of the study. The phase angle differences had a spread of 3 minutes only.

A statistically signigicant linear relationship was established between ;the timing of emergence ofthe first bat and the time of sunset (r=0.99;P<0.01).

3.2 Return activity.

The timing of end of activity (the time when the last bat returned) precisely preceded the timings of sunrise , irrespective of the month of study. A statistically significant correlation was obtained between the timing of return of the last bat and the time of sunrise (r=0.99;P<0.01). Phase angle differences were computed taking into consideration the median timing of return activity in relation to respective timing of sunrise. The spread of phase angle differences amounted to 19 minutes.

3.3 Activity time

Results of correlation analysis revealed a statistically significant linear relationship, between the length of activity period and night length (r=0.97;P<0.01).

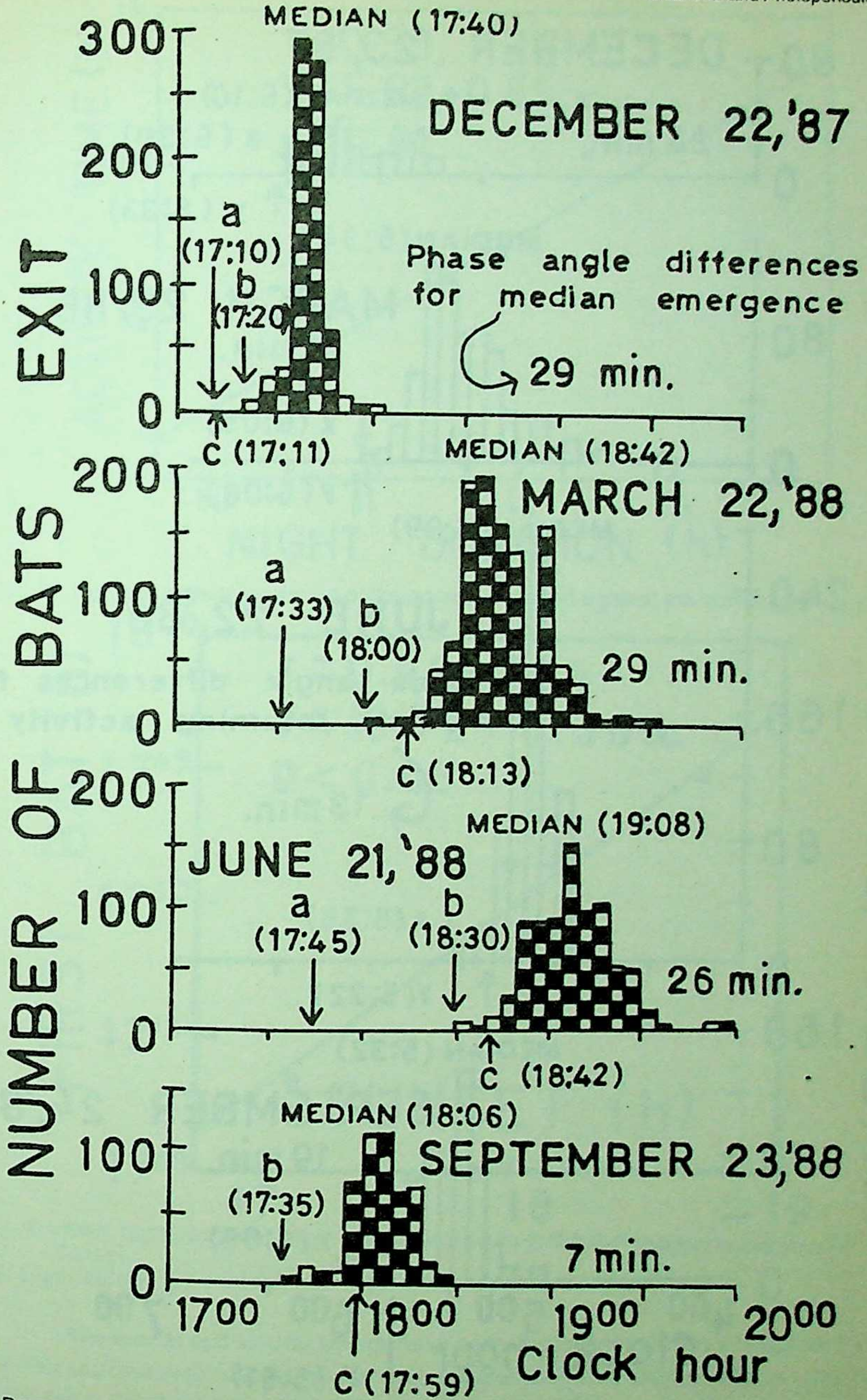

Fig. 1: Representative example of the pattern of emergence flight activity of bats, for four different months. Bar represents the total number of bats leaving the cave over five minutes. The arrow marking a b c in each coloumn indicates the time of pre- emergence activity, emergence of first bat and timing of sunset respectively.

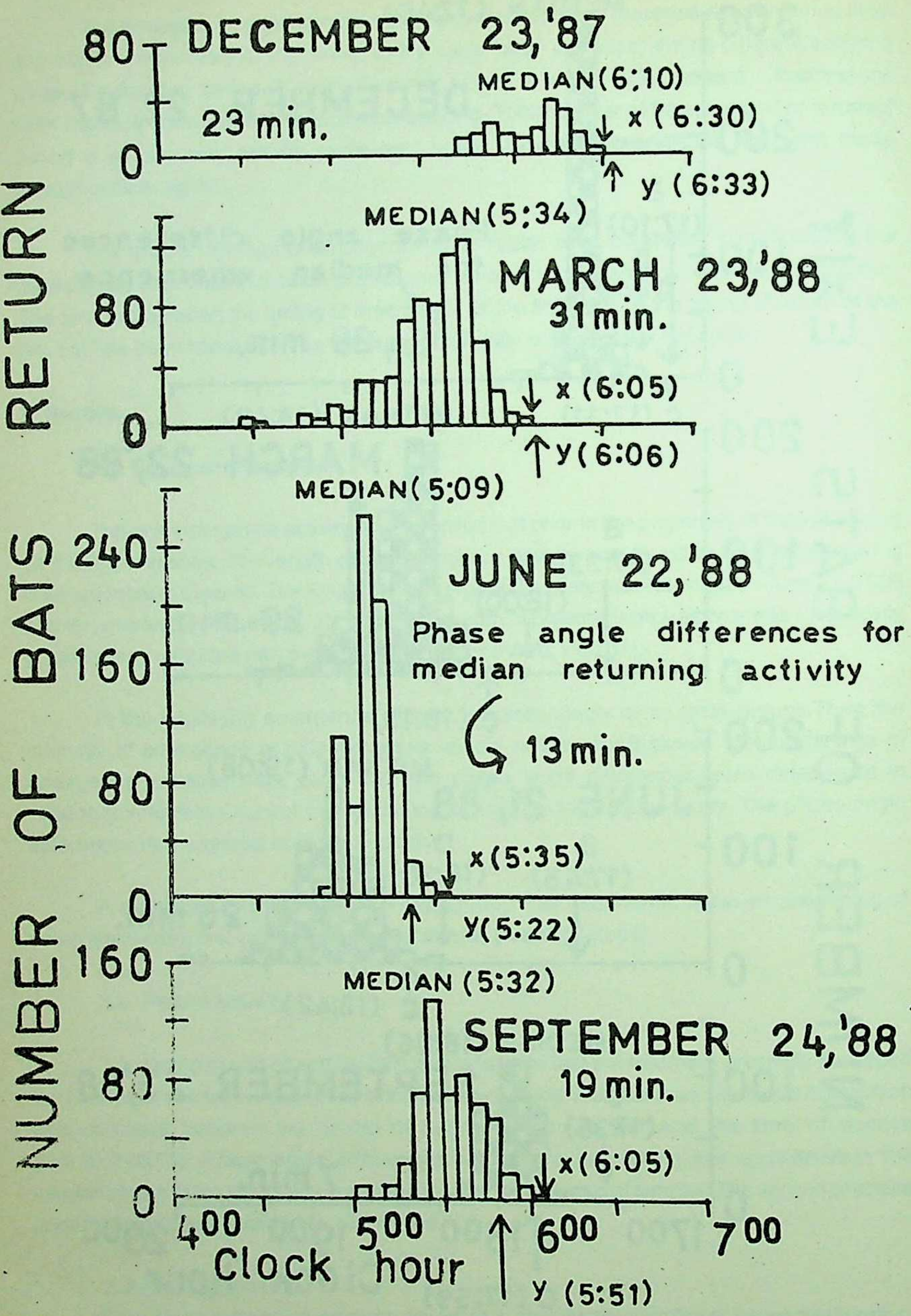

Fig. 2: Representative example of the pattern of returning activity of bats. for four different months. Bar represents the total number of bats entering the cave over 5 minutes. The arrow marking x, y. in each column indicates the time of returning of last bat and time of Sunrise respectively

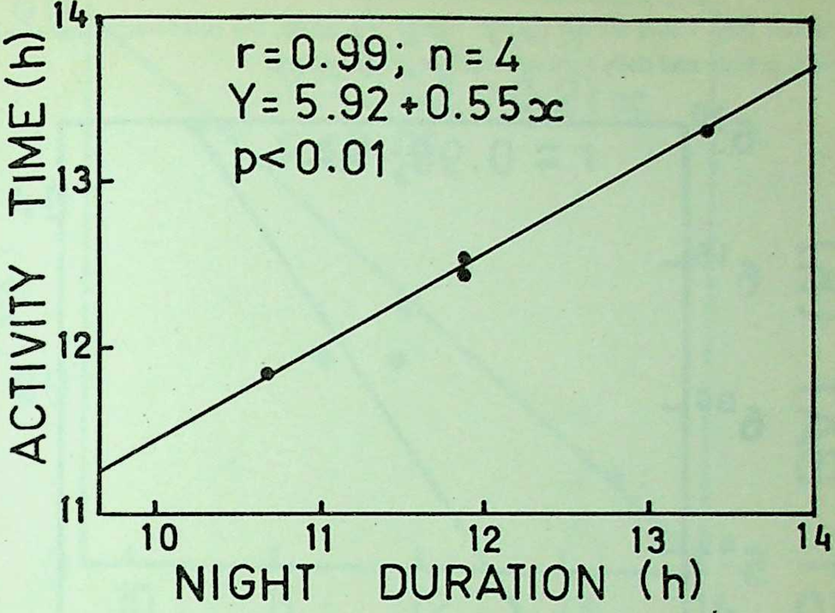

Fig. 3: Duration of activity period of the colony is plotted against the duration of night time.

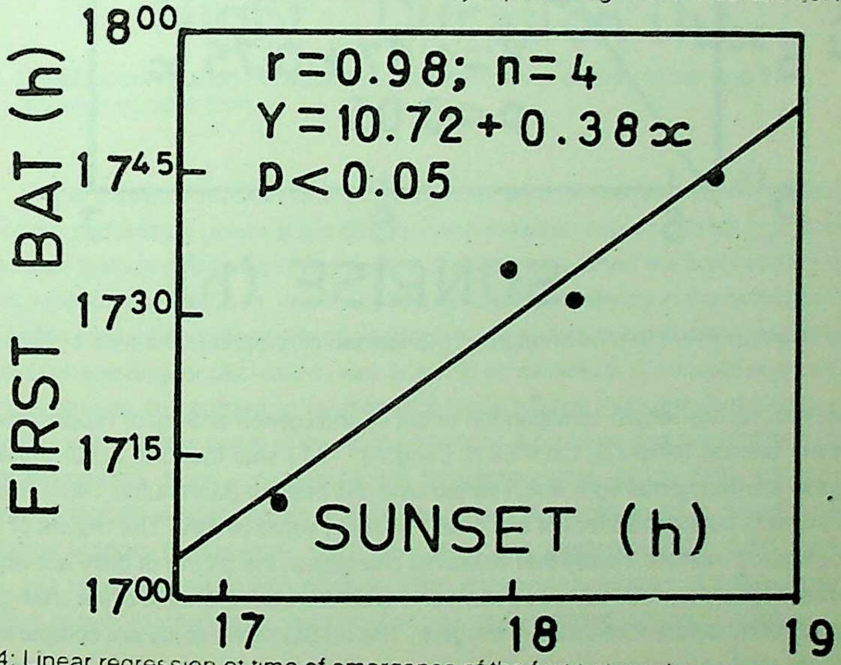

Fig. 4: Linear regression of time of emergence of the first bat in relation to the time of Sunset

4. Discussion

The sunset and sunrise timings have been implicated to act as a zeitgeber for onset and for end of activity, respectively, in nocturnal animals (Prittendrigh and Daan 1976; Nuboer et al. 1983; Takahashi et al. 1984). the bats are nocturnal mammalsand depend more upon auditory rather than visual stimuli (Griffin 1958). Therefore, the relationship between bats exodus flight activity and daily light cycle are of great interest.

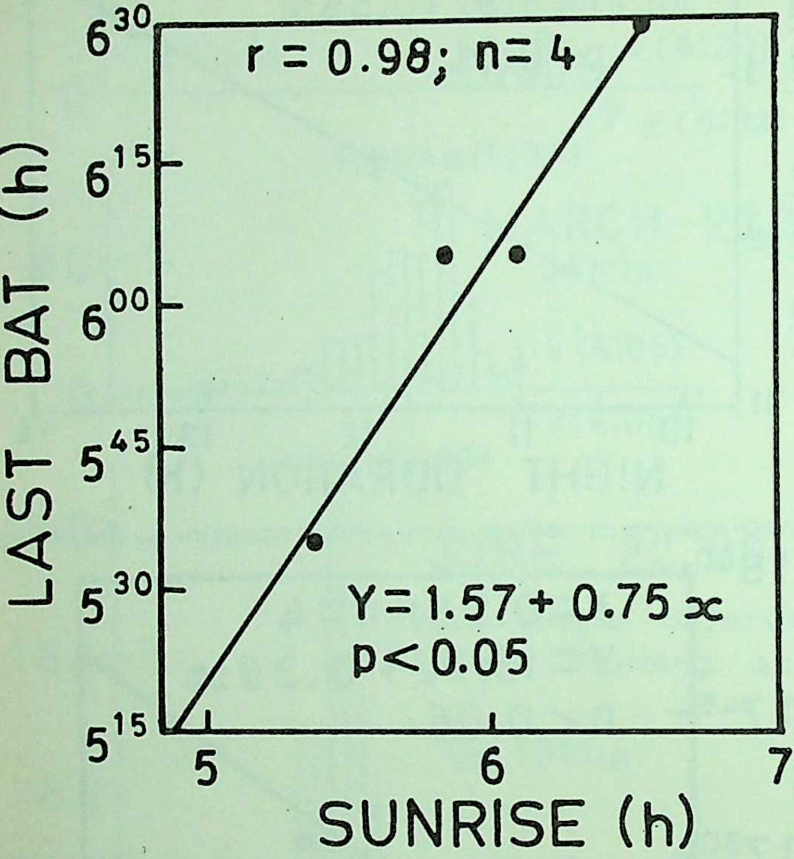

Fig. 5 Linear regression of the time of return of the last bat in relation to the time of Sunrise

Under natural day length condition the onset of emergence activity in Hipposideros speoris at lower latitude (Madurai; Lat.9°58'N; Long.78° 10'E) was found to occur within a certain range fo environmental light which varied over the season (Marimuthu,1984). However, it was found to be independent of any fixed threshold value of light. The results of the present investigation indicate clearly that seasonal changes in the timing of daily activity in this tropical bat Hipposideros fulvus keep pace with the different components of the changing light dark cycle during different seasons of the year. The results of the study are comparable with those conducted on H. speoris at Madurai (Marimuthu, 1984).The time lag between the onset of pre-emergence activity and the emergence timing was found to be different during different seasons fot the year for H. fulvus at this latitude (Lat. 18°52'N; Long.82°56'E). The length of this time lag was greater during summer when the night duration was shorter, and the same time lag become shorter during winter when the night duration was longer. Could there be a definite threshold of light intensity which triggers the emergence of bats ?

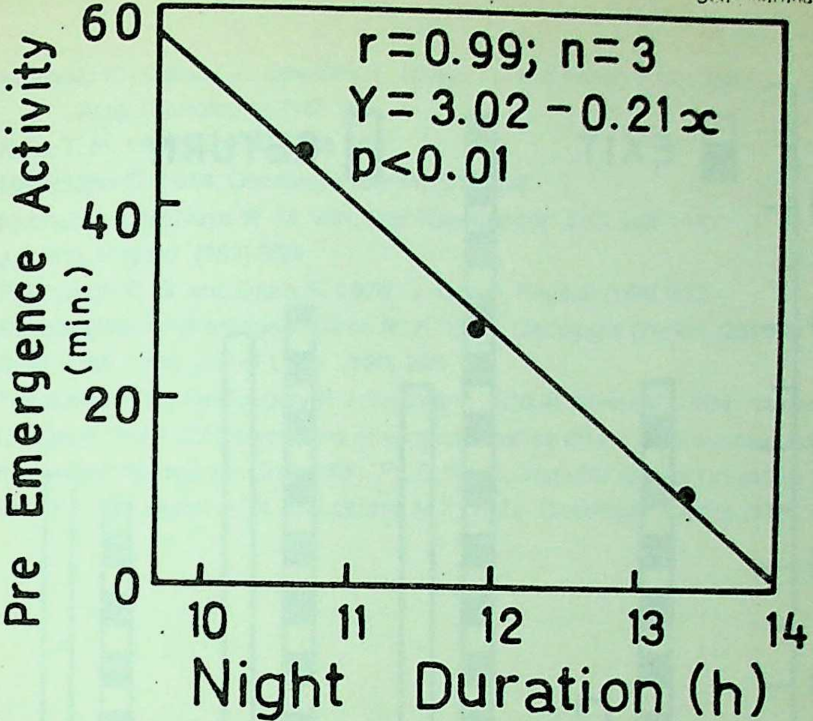

Fig. 6 Negative correlation between the pre-emergence activity of bat and the duration of night time.

In this study the median timings for both the emergence and returning activities have been computed and the phase angle differences have been determined taking into consideration median timings of these activities rather than the time when the first bat emerged or the last bat entered into the cave. In our opinion the central tendency in the behavioural pattern of the entire population reflects the characteristic of a circadian oscillator more precisely. The spread of phase angles calculated , was found to be mimimum (3 minutes only) with respect to sunset timings. In contrast, in relation to sunrise timings the spread of phase angle was relatively more . On the basis of these observations it could be concluded that probablly the sunset timings are more important for the Synchronization of the daily activity rhythm of H.fulvus. Although similar pattern of emergence has been observed in different bat species by several authors (Davis et al. 1962; Jacobson and Duplessis 1976; Swift 1980; Usman 1981). yet none has taken into consideration the median timings of either emergence or returning activity for the computaation of phase angles during different seasons of the year.

Bats inhabiting temperate zone caves have been known to undergo hibernation (Dulic 1962; Frick and Felten 1952; Kahman and Goenner 1956; Kowalski et al. 1986). Usually the temperature of any cave refllects the annual mean temperature of the particular area where the cave is located (Barr 1968). Results of the present study clearly reveal that H.fulvus does not undergo hibernation. However, the duration of activity during winter months was found to be greatly reduced. A marked difference in the intensity of emergence and return during

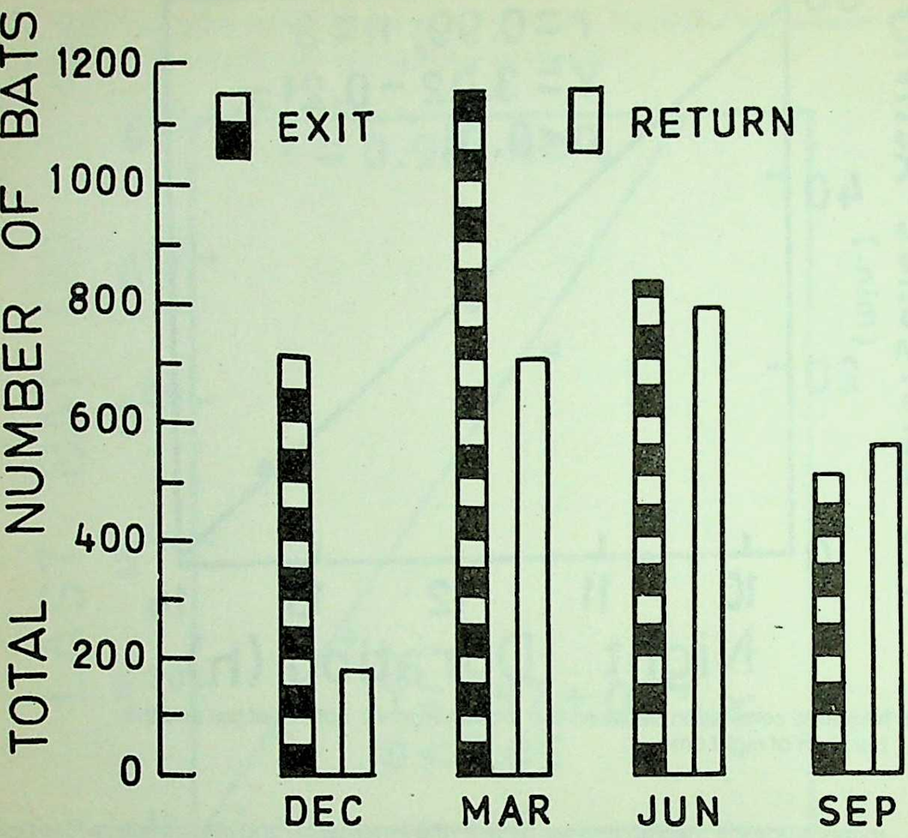

Fig. 7: Histograms representing the total number of bats emerging from the cave as well as returning back to the cave, at four different months of a year.

December suggests that the time window for return is relatively, wider and spreads all over the night length. It shows that the external temperature may be unfavourable for activity of bats during winter but not bad enough for total suspension of activity.

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Effect of Rhizobium and Root Knot Nematodes on nitrogen fixation and nitrate utilization in Chickpea (Cicer arietinum L.)

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Ms: Received 21.5.90; Revised 5.5.93.

Abstract. Effect of Rhizobium spp. and rot-knot nematodes, i.e. Meloidogyne incognita on N_2 fixation as well as absorption and translocation of N was studied in chickpea plant (Cicer arietinum L.) cultivar G-130 grown in earthen pots containing steam sterilized sandy loam soil. N in the form of KNO_3 was applied before sowing @ 10 ppm. M. incognita caused significant reduction in nodulation, nitrogenase activity of nodules and nitrate reductase activity of leaves.

Key words. Rhizobium sp., Chickpea, Meloidogyne incognita, Nitrogen fixation.

1. Introduction

Pulses being leguminous plants fulfil their nitrogen requirement through symbiotic N_2 fixation. In addition, they can make use of nitrogenous fertilizers applied to the soil. However, application of high doses of NO_3 inhibits N_2 fixation (Sawhney et al., 1985). Pulses are known to be parasitized and damaged by Meloidogyne incognita (Chahal and Singh, 1983) which adversely affects N_2 fixation and uptake of plant nutrients (Chahal and Chahal, 1987). The present communication reports the effect of Rhizobium spp. and M. incognita on N_2 fixation and NO_3^- utilization by chickpea plants.

2. Materials and methods

2.1. Inoculation with Rhizobium spp. and nematodes :

Healthy viable seeds of chickpea (Cicer arietinum L.) cultivar G-130 (10g) were surface sterilized, before sowing (Vincent, 1970). For Rhizobium inoculation 4-day-old culture of Rhizobium sp. strain R-6 isolated from the nodule of chickpea plant were used to inoculate the seed. Rhizobium culture was maintained on the slants of yeast extract mannitol agar medium. Initially, 5 seeds were sown/pot but later on thinned to one seedling/pot. One week-old seedlings raised from inoculated with freshly hatched juveniles of M. incognita. For this, 10 ml suspension containing 2000 juveniles/pot (optimum damaging level) was poured uniformly all over the exposed root seedlings which were later on covered immediately. Culture of M. incognita was maintained on tomato plants grown in earthen pots containing steam sterilized soil.

2.2 Pot Culture experiment :

Pot culture experiment was conducted in earthen pots (15" diameter) containing 750 g steam sterilized sandy loam soil poor in nutrients (pH=8.5, N=0.5%, P=5 ppm and K=8 ppm). Experiment comprised 4 treatments, namely control, Rhizobium sp., M. incognita and Rhizobium sp.+ M. incognita. each treatment had three replications. Nitrate in the form of KNO_3 was applied @ 10 ppm to each pot before sowing. Nitrate reductase activity of leaves was measured after 40 days of sowing (Jaworski, 1971). Nitrogenase activity of nodules was measured after 45 days of sowing (Hardy et al., 1973). N of shoot was estimated by Micro Kjeldahl method.

3. Results and discussions

Nematode infection resulted in decrease in mumber, dry weight and nitrogenase activity of nodules, dry weight and N content of shoot. Nitrogenase activity of nodules was also affected (Table 1). It is obvious from the results that the infection of root system by M. incognita lead to significant decrease in all the above mentioned growth characters as compared with Rhizobium treatment without nematodes. There are other reports also regarding reduction in the nodules and N_2 fixation due to mematode infection in the legume crops (Sharma and Sethi, 1976; Chahal ,1983; Chahal and Singh, 1984). The reduction in nodulation and N_2 fixation may be due to nutritional deficiency (Masefield, 1985) and physiological changes in the infected plants (Sharma and Sethi, 1976). In addition,

Table 1: Effect of Rhizobium sp. and Meloidogyne incognita on the nitrogen fixation and utilization of nitrate by chickpea (Cicer arietinum L.)

Treatments	Number of nodules/ plant	Nitrogenase activity (μM C_2H_4/h/ g of nodules)	Dry weight of shoot (mg)	N content of shoot (%)	Nitrate reductase activity of leaves (μ M NO_2/h/g leaves)	Number of galls/plant
Control	0	0	700	1.8	4.21	0
Rhizobium	60	24.57	795	2.1	4.21	0
Nematodes	0	0	670	1.5	4.07	16.8
Rhizobium+ nematodes	40	18.70	650	1.4	3.75	17.3
C.D. at 5%	3.869	2.903	30.57	1.93	0.796	4.172

nematodes interfere with the development of nodules as they are known to excrete hydrolytic and oxidative enzymes which could play determinative role in the inhibition of nodule development (Baker et al.,1972). Nematodes not only reduced nodulation but also decreased dry weight of shoot which may be attributed to decreased N_2 fixation (Chahal and Chahal, 1988a). Per cent N uptake also decreased due to namatode infections compared with Rhizobium treatment without nematode infection as compared with Rhizobium treatmentwithout nematodes. Maximum N content of shoot was observed in Rhizobium inoculated plants. There was no significant difference in the number of galls produced in the presence or absence of Rhizobium.

Nitrate reductase activity (NRA) of leaves was maximum in case of uninoculated control. The leaves of M. incognita inoculated plants showed decrease in the NRA than that of all other treatments probably due to interference in the translocation of nitrogen from root/nodules to the above ground parts of plants. The results support the idea that nitrogen might have accumulated in the root/nodules and its limited amounts were translocated to the shoots of infected plants Chahal and Chahal, 1988b).

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Studies on the Sorption Behaviour of the Amino Acids, Arginine and Histidine on Chemically- Pretreated Alumina in relation to Chromatography.

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Abstract. The sorpotion- desorption behaviour of arginine (Arg) and histidine (His) from 90% ethanol on $HClO_4$/ CH_3COOH- treated alumina of surface hase pH 3.0-9.0 is described. The elution behaviour and column chromatography were also investigated to correlate the data. The sorption of Arg and His was found to depend on the pH of the substrate as well as on the acid used for pretreatment of the oxide. The sorption was found to increase in the pH range 4.0-9.0. The process was moderately fast (between 10 min and 24 hr) and almost athermic (20°-50°C). Desorption efficacy and retarding influence on adsorptionby cations was in the order : $Th^{4+} > Al^{3+} > Mg^{2+} > Na^{+}$. The distribution coefficlent for adsorption (K_d) and desorption (K_D) were determined , and on the basis of these studies, the elution behaviour and column chromatography was carried out.

Key words, Sorption, desorption, amino acid, arginine, histidine, alumina.

1. Introduction

The adsorption of amino acids on metal oxide particles has received increasing attention over the last few years (Kumanomido et al 1978; Nakata et al 1978; Okazaki et al 1981; Korana and Sugier 1980; Mitsubishi Rayon Co. 1983 and Fuerstenau et al 1984). The cation-exchange adsorption of some amino acids on Na, Mg, Cs, Co, Ni and Cu-forms of montmorillonite was studied at pH 2 by Tarasevich et at (1979)- and Rak et al (1981). The sorption capacity was found to increase with increase in their basicity. The study of the adsorption of-amino acids at their isoelectric point onto montmorillonite saturated with mineral cations, showed that divalent cations have greater affinity for the amino acids than the monovalent cations (Siffert and Kassaissia 1978). Kijami et al (1981) suggested hydrogen bonding and involvement of phosphate sites of zirconium phosphate to the- NH_2 groups of amino acids. Tentorio and Canova(1989) have investigated the adsorption of L- glutamic acid, L-(+)-lysine and DL-serine on amorphous titanium hydrous oxide as a function of pH. They suggested hydrogen bonding between deprotonated amino groups and surface hydroxyl groups as a possible explanation for observed adsorption, when electrostatic repulsion prevails.

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Both metal oxides and amino acids have a charge which is pH-dependent, making it possible to evaluate adsorption under different electrostatic conditions. Furthermore, the surface of metal oxides has acid/ base sites, which may react with functional groups of the amono acid. Finally, the adsorption of amino acids represents a first step in the understanding of the complex aspects of protein adsorption, even- though it cannot account for the entropic contribution, which is important when polymeric solutes are involved (Lyklema 1984 and Norde 1986).

In some of the earlier studies (Moitra et al 1984, 1986,1988, 1989 and Parminder et al 1989, 1990), the role of acid and alkali pretreatment of alumina, in the adsorption of amino acids has been shown. This work deals with the sorption- desorption behaviour of Arg and His on $HCLO_4/CH_3COOH$- treated alumina of varying surface-phase pH from 90% ethanol in water. As alumina has no affinity for the amino acids in water, and since they are sparingly soluble in ethanol, the studies were carried out in 90% ethanol as slovent. The elution behaviour and column chromatography were also investigated to correlate the data.

2. Experimental

$HClO_4/CH_3COOH$- treated alumina samples (to be designated as Al_2O_3(p) and Al_2O_3(a), respectively of pH 3.0 to 9.0 were prepared by treating (24hr, 25g) Brockmann alumana (S.Merch, activity: Grade II, predominant particle size: 200 mesh, pH9.5) with varying concentrations of perchloric acid (0.002-0.25N) and acetic acid (0.005-2.0N), as described earlier (Parminder et al 1989). The surface area of some of the samples was determined by the "standard continuous flow method". Hydroxide ion-exchange capacity of the samples was determined by the pH- titration method. The anions (ClO_4- or CH_3COO^-) present on the acid- treated samples were estimated (Bebko and Pilipenko 1976; Snell and Snell 1967), and the results are shown in table1.

Arginine and histidine (E. Merck) werefurther purified by crystallization from ethanol water mixture (1:1, v/v)and dried at 105°C. From the stock solution of the solutes in 90%ethanol (2.00 m mole dm^{-3}), 5-80 cm^3 of the solutions were withdrawn, diluted (100 cm^3), and estimated colorimetrically at 570 nm. (Ninhydrine method, Lamothe and McCormick 1972) using a spekol Spectrocolorimeter. The pH of the substrate samples was measured in 90% ethanol solution (1% extract) as well as in an aqueous medium for comparison (table1). The pH of the electrolytes (in 90% ethanol) used as descorbents was also measured. The solubility of the amino acids in 90% ethanol was 0.10 (Arg) and (0.08 (His), While the solubility in the aqueous medium was 16.0 and 5.40 g per 100 cm^3 at 30°C, respectively.

The adsorption study on alumina (0.10 g)was conducted from 90% ethanol solutions (10 cm^3) at different time intervals, temperatures and in presence of various inorganic electrolytes. The desorption study was conducted with 90% ethanol solutions of the same electrolytes. The other experimental details were the same as reported earlier (Moitra et al, 1989).

Table-1: Anion content, OH^- -ion exchange capacity and surface area of Acid - treated Alumina samples of varying surface-phase pH.

Predominant grain size : 200 mesh; Surface area: 80 ± $3m^2g^{-1}$
Pore-size distribution :2 5-30 A^o; Surface-phase pH:9.5

Surface-phase pH of alumina		Anion Content (mg ion g^{-1})		OH-ion exchange capacity		Surface area (s ± 3 m^2 g^{-1})	
in aq. medium	in 90% ethanol	ClO^{-4}	CH_3COO^-	Al_2O_3 (p)	Al_2O_3 (a)	Al_2O_3 (p)	Al_2O_3 (a)
3.5	4.0	0.191	-	0.80	-	90	-
4.0	4.5	0.170	0.268	0.63	0.25	90	70
4.5	5.0	0.102	0.196	0.56	0.20	87	73
5.0	6.0	0.074	0.152	0.48	0.16	90	76
5.5	6.3	0.036	0.138	0.45	0.13	86	79
6.0	6.7	0.027	0.120	0.43	0.10	89	80
6.5	6.9	0.023	0.104	0.41	0.08	-	-
7.0	7.2	0.019	0.088	0.40	0.05	87	82
7.5	7.3	0.017	0.080	0.36	0.03	-	-
8.0	7.5	0.013	0.068	0.33	0.02	90	85
8.5	8.4	-	-	-	-	-	-
9.0	8.6	-	-	-	-	-	-

Al_2O_3(p) and Al_2O_3(a) designate $HClO_4$ and CH_3COOH- treated alumina, respectively.

3. Results

The variation of adsorption as a function of time (10 min-24 hr, 30 ± 1°C, pH 9.0, 2.00 to 0.10 m mole dm^{-3}) indicates the process to be moderately fast. Thus, in 10 min, Arg shows 13 to 57% [Al_2O_3(p)] and 40 to 67% [Al_2O_3(a)] adsorption, while for His the respective values are 33 to 45% and 33 to 40%. Further, in the pH range 4.0-8.0 (2.00 m mole dm^{-3}) 49 to 56% [Al_2O_3(p)], 25 to 54% [Al_2O_3(a)] for Arg and 21 to 35% [Al_2O_3(p)], 9 to 26% for His, adsorption is observed after 10 min contact. However, the equilibrium is established in 4-6 hr.The temperature effect studies (20°- 50°C, 4hr, pH 4.0 - 8.0) are found to be almost athermic.

The results of adsorption versus pH areshownin Figs. 1&2. The adsorption is found to be pH dependent; the adsorption increases in the pH range 4.0 -9.0 for Al_2O_3(p) and Al_2O_3(a) in case of the amino acids. The affinity vanishes at pH 3.5.

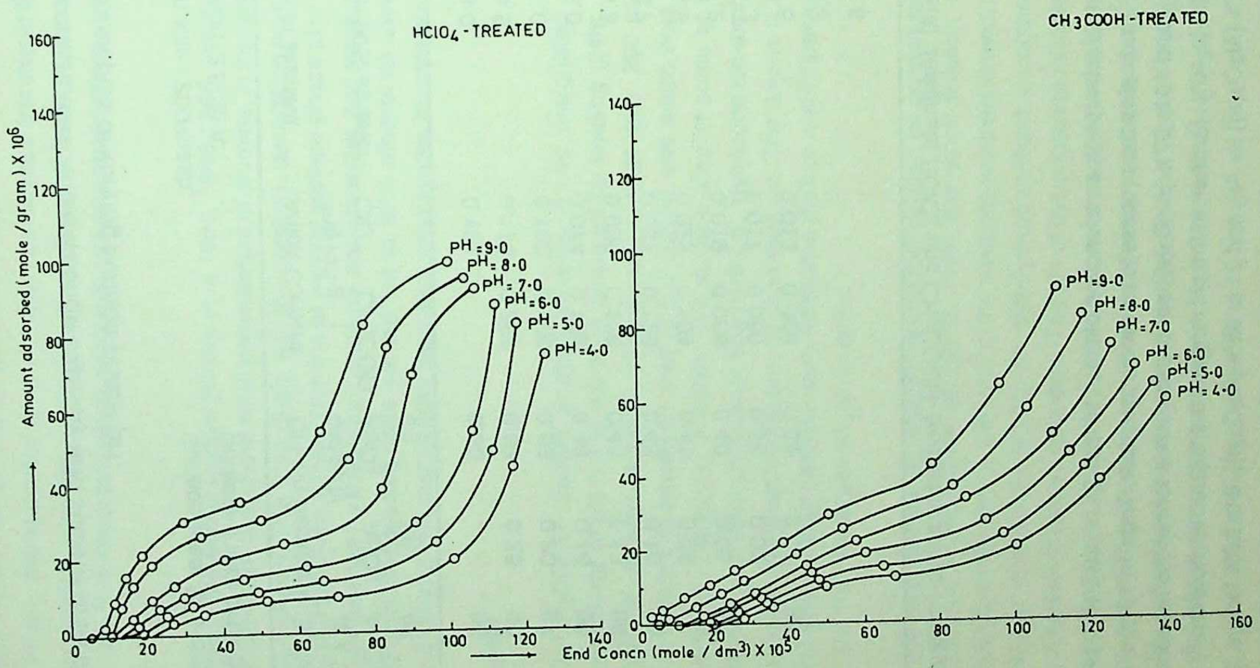


Fig. 1: Change in the nature of adsorption of Arginine from 90% ethanol solutions on acid treated Alumina of varying pH (Time : 24 hr; Temperature : 30 ± 1°).

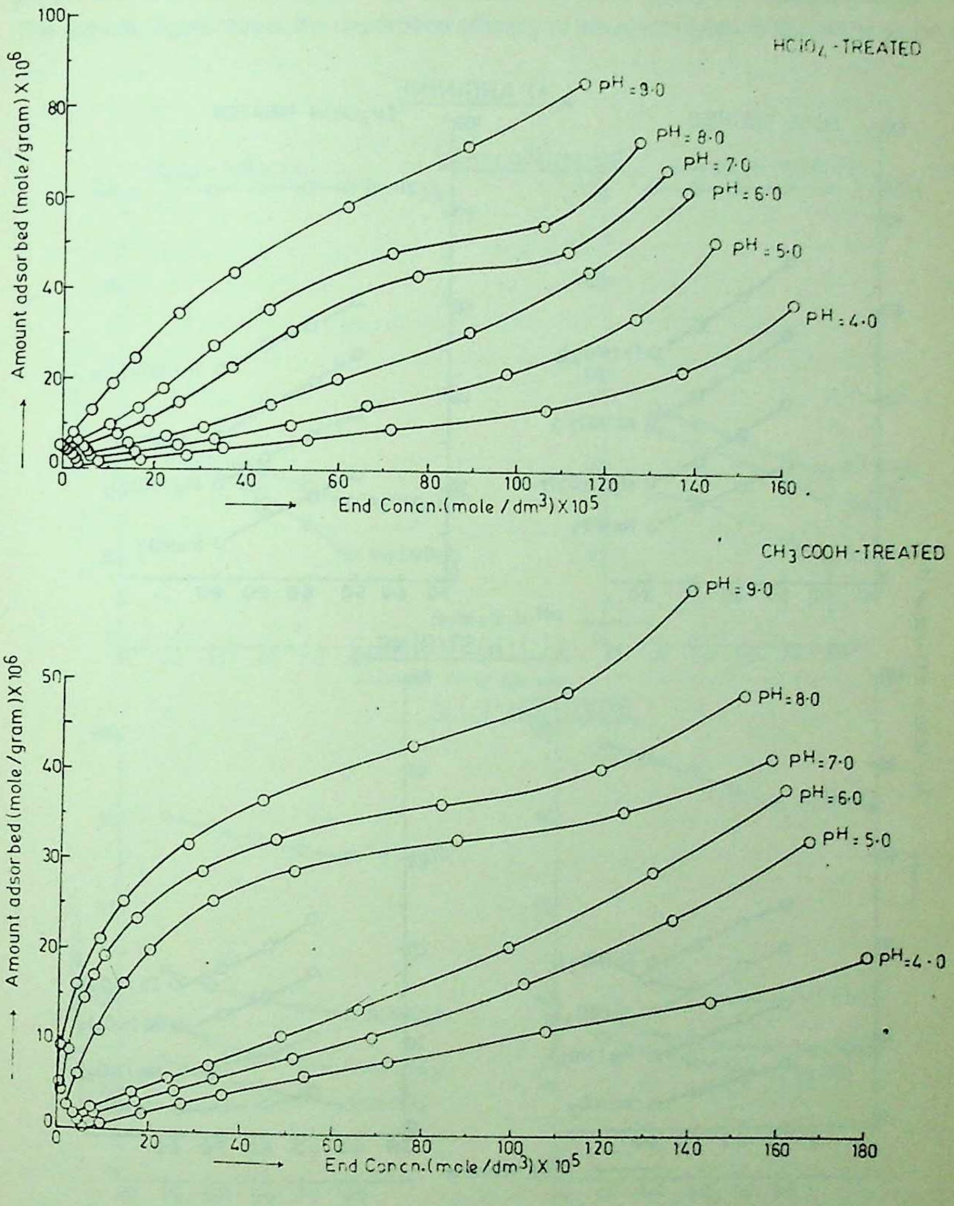


Fig. 2: Change in the nature of adsorption of L (-) Histidine from 90% ethanol solutions on acid treated Alumina of varying pH (Time : 24 hr; Temperature : 30 ± 1°).

Adsorption studies in presence of electrolytes (Nitrates of Na^+, Mg^{2+}, $Al^{3+}$and Th^{4+}) were also carried out (30 ± 1°C, 24hr). The adsorbate and the electrolyte concentrations were kept the same, 2.00 0.80 and 0.20 m mole dm^{-3}. The results (Fig.3) reveal a significant decrease in adsorption, and the retarding influence is in the order Th^{4+}> Al^{3+} > Mg^{2+}> Na^{+}.

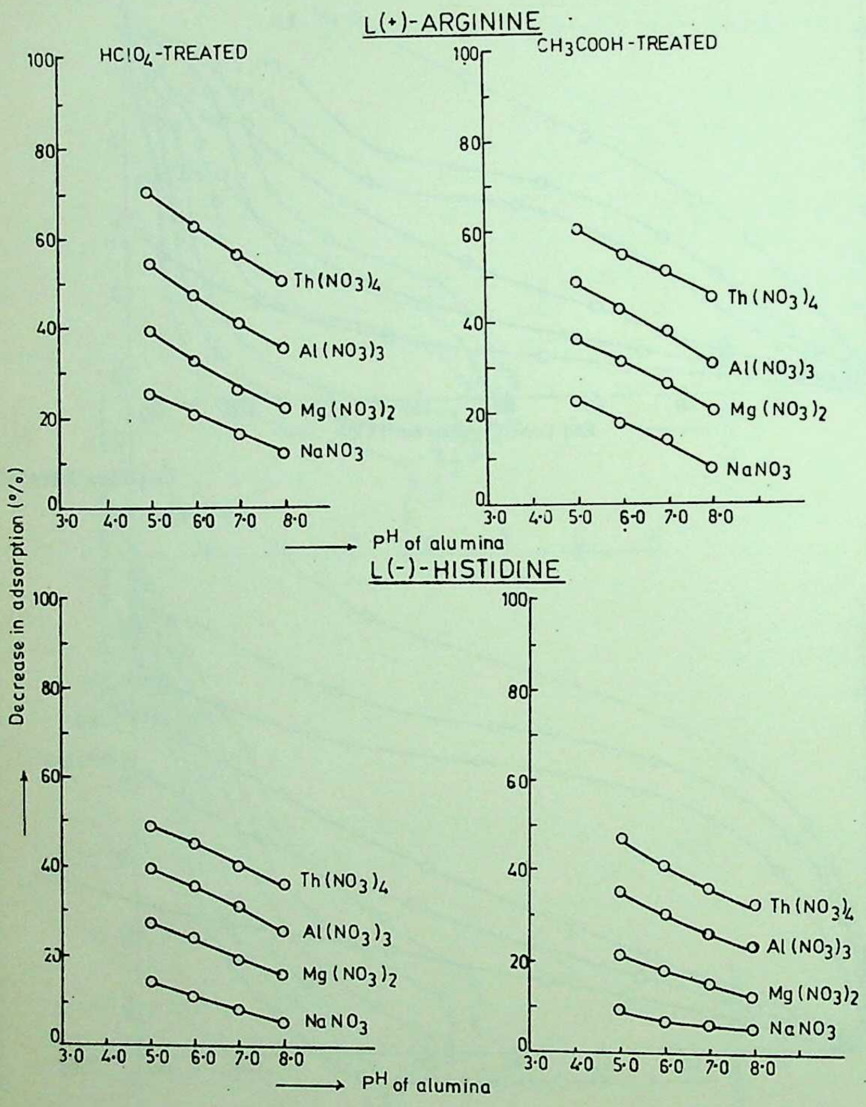


Fig. 3: Change in percentage decrease in adsorption of basic Amino acids from 90% ethnol solutions in presence of electrolytes with varying pH of Alumina. (24hr, 30°, [Amino acid]o β [Electrolyte]o = 2.00 X 10^3 mole/dm^{-3}).

Desorption of the amino acids adsorbed on alumina (pH 4.0 - 8.0, 30 ± 1°C, 24hr) was examined with the above clectrolytes in 90% ethanol (0.001 & 0.01M) as a function of pH of the substrate and concentration of the amino acids (2.00, 0.80 and 0.20 m mole dm^{-3}). The results (fig.4) reveal the desorption efficacy of the electrolytes to follow the same trend.

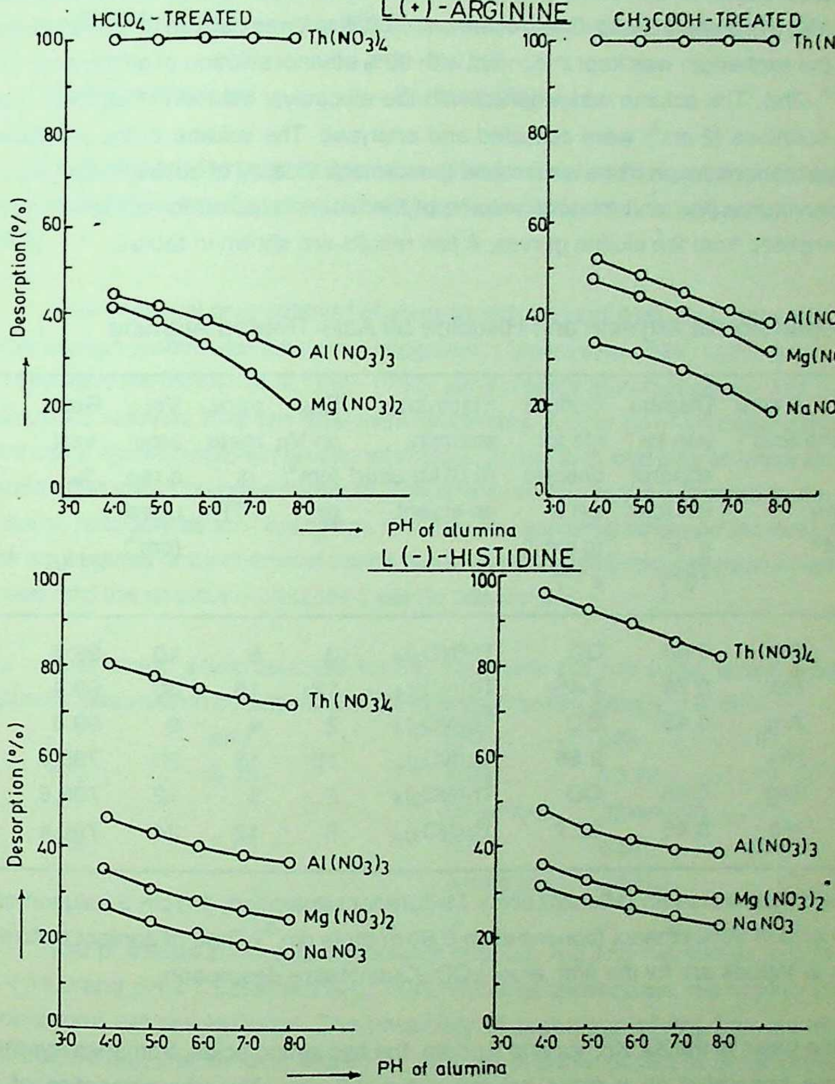


Fig. 4: Change in percentage desorption of basic Amino acids absorbed on Alumina of varying pH from 90% ethanol solutions by electrolytes solution (30 ± 1°, [Amino acid]o = 2.00 X 10^{-3} mole/dm^{-3}, [Electrolyte]o = 0.01M)

The degreeof desorption increases with increase in concentration of the electrolytes. The extent of desorption is found to depend on the pH of the substrate as well as the acid used for the pretreatment it decreases with increase in pH of alumina (pH 4.0 to 8.0).

From the adsorption and desorption studies (batch operation), the distribution coefficients for adsorption (K_d, cm^3g^{-1}) and desorption (K_D, g cm^{-3}) were also detemined (Moitra et al 1989). On the basis of these data, the elution studies were performed with 90% ethanol solution (0.01M) of the electrolytes. Glass column (10 x 0.5 cm) was packed (dry) with alumina (1.0 g), and the exchanger was kept in contact with 90% ethanol solution of amino acid (0.80 m mole dm^{-3}, 2hr). The column was washed with the electrolyte solution (1 cm^3min^{-1}) and the effluent solutions (2 cm^3) were collected and analysed. The volume of the eluate was plotted against concentration of the amino acid (percentage elution) to obtain elution curves. The retention volume (V_R) and the total volume of the eluent required for complete elution (V_T)were computed from the elution curves, A few results are shown in table 2.

Table-2: Separation of Arginine and Histidine on Acid-Treated Alumina

Sepn. No.	pH of alumina and column height* (cm)	Amino acid	Distribution for adsorption (cm^3 g^{-1}x 10^{-2})	Cofficients for desorption a (g cm^{-3}) x 10^2	Electroyte solution (0.01M) used as eluent	Elution V_R (cm^3/g)	parameters VT)	Volume a required (cm^3)	Recover(%
a	8.0	Arg	0.60	QD	$Th(NO_3)_4$	3	6	10	99.7
	6.0	His	0.78	2.45-	$Th(NO_3)_4$	12	18	30	99.4
1.	7.0	Arg	0.43	QD	$Th(NO_3)_4$	2	4	8	99.8
	5.0	His		2.86	$Th(NO_3)_4$	10	13	20	799.5
2	8.0	Arg	0.48	QD	$Th(NO_3)_4$	2	5	12	799.6
	8.0	His	0.45	12.7	$Th(NO_3)_4$	8	12	24	799.4

* Quickfit glass cloumn (10cmx0.5cm); Mixture for separation; 1.0 cm 3 solution of each amino acid in 90% ethanol (conentration 0.80 m mole dm^{-3});Time of contact before; elution, 2 hr;a- Values are for the first eluent;QD- Quantitative desorption.

On the basis of the K_d, K_D, V_R and V_T data, the two amino acids, from their synthetic mixture, were separated on a glass column (10 x 0.5 cm). The characteristics of the separation are shown in table 2.

4. Discussion

The adsorption of organic solutes from aqueous solution may be influenced by the hydrogen-ion concentration in a variety of ways. The most obvious effect in the speciation of the weak polyprotic acids, which occurs as a function of pH. This effect may be observed as a surface preference for one particular species, or as a pH- dependent increase or decrease in electrostatic attractive or repulsive forces due to the differing overall charge on each acidic species. However, the oxide surface also exhibits acid- base characteristics, giving rise to various charged groups on the surface, such as > $AlOH^+_2$, $AlOH^o$ and AlO^-, as per Eqn. (1) (Parks and Bruyn 1962).

$$> AlOH^+_2 \underset{+H^+}{\overset{-H^+}{\Longleftarrow\!\!=\!\!=\!\!=\!\!=\!\!\Longrightarrow}} AlOH^o \underset{+H^+}{\overset{-H^+}{\Longleftarrow\!\!=\!\!=\!\!=\!\!=\!\!\Longrightarrow}} AlO^- \qquad (1)$$

Surface at low pH Surfaceat high pH

Not only are these species responsible for the electrical potential exhibited by oxide particles, but they may also exert differing preferences for organic solutes (Bowers and Huang 1985).

The chemical pretreatment of alumina with an acid (HX), as a means of activation of anion exchangeable sites has been suggested (Moitra et al 1984, 1986, 1988, 1989; Kar et al 1988, and Parminder et al 1989, 1990). Amino acids are ampholytes, and at neutral pH, in aqueous solution, they can exist as polar species. A Change of pH beyond their isoelectric point has a significant effect on the net charge. This in turn, can play an important role in their sorption behaviour on polar sorbents. The amino acids may be adsorbed by the acid treated alumina, probably by ion- exchange mechanism, involving single amino acid molecules or their aggregates or by chemical bonds, such as hydrogen bond, between substrate surface groups and the sorbate molecules (specific adsorption).

The amino acids selected for the study are the two basic amino acids with their apparent dissociation constants (25°C) as shown below (Morris, 1978):

	pK_1	pK_2	pK_3	PI
Arg	2.18	9.09	13.20	11.15
		(α-amino)	(guanidyl)	
His	1.78	5.97	8.97	7.47
		(imidazoyl)	(NH_{3+})	

The pI values show that in aqueous medium Arg and His should exist as cations at pH < 11.0 and pH < 7.5,respectively. Thus, aluminacannot retain His at pH > 8.0, since the anionic form will predominate. The possibility of adsorption of Arg from aqueous solution acid-treated alumina surface. Moreover, alumina is known to act as an anion exchanger at pH ≤ 8.0 and as a cation exchanger above this pH (Abe and Ito, 1965). Because of the two opposing effects of pH on both the substrates and solutes in an aqucous medium, adsorption affinity is absent.

However, the results show that the sorption of the basic amino acids on alumina from 90% ethanol is pH- dependent and it increases with increasing pH (pH4.0-9.0). To explain the variation of adsorption the following observations may be of significance :

(1) The pH of alumina samples in 90% ethanol ranges between 4.0-8.6, instead of 3.5-9.0 in an aqueous medium.

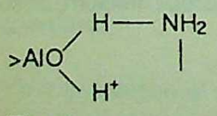
(2) The pH of solutions of Arg and His (2.00 m mole dm^{-3}) in 90% ethanol is 9.5 and 7.2 respectively.

(3) It is known that in 90% ethanol the isoelectric pH of these basic amino acids is lowered from their original value [pH11.15 (Arg) and pH 7.47(His], and so they are likely to carry a positive charge in the pH range 4.0-8.6 (Ledererand Lederer1967.)

(4) The Solubility of these amino acids in 90% ethanol is much less than that in an aqueous medium (the values have been given earlier). Moreover, the solubility of an amino acid is affected by the pH. The Zwitter ionic form will be the least soluble form (Mark and Nickoles 1981).

Thus, the adsorption behaviour of these amino acids with respect to pH (pH > 8.0) may be explained as per Eqn (1) and in this pH range they appear to be adsorbed primarily by cation exchange. With a progressive decrease in the pH of alumina, the surface is expected to acquire an increasingly positive charge (Eqn.1). This in turn will repel the amino acid species (cationic),decreasing the adsorption gradually, so much so that it drops to zero at pH < 3.5 However, the electrostatic repulsion for Arg and His seems to be high enough to prevent a close approach of the amino acids to the substrate. Thus, a simple electrostatic consideration, i.e. the chances of adsorption of the amino acids by ion- exchange, cannot explain the observed up take of the solutes in the pH- range 4.0-8-0.

Morrison (1984) observed adsorption of negatively charged species onto oxides at pH above the i.e.p.in case of α - hydroxy acids. He explained the uptake in terms of hydrogen bonding betwen α - hydroxyl groups of the acid and the negatively charged oxide ions on the surface. A similar mechanism was assumed for the adsorption of α - amino acids on amorphous hydrous oxide, when electrostatic repulsion prevails (Tentorio and Canova1989).The adsorption of various organics onto hydrous oxides has often been attributed to hydrogen - bond formation (Giles 1959;Pimentel and McClellan 1980). A hydrogen bond may form between two functional groups, one serving as a proton donor or Bronsted acidic group and another group serving as an electron donor or Lewis basic group, as shown below :



Type1
(proton donor)

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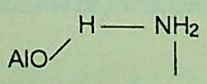
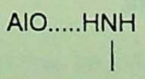


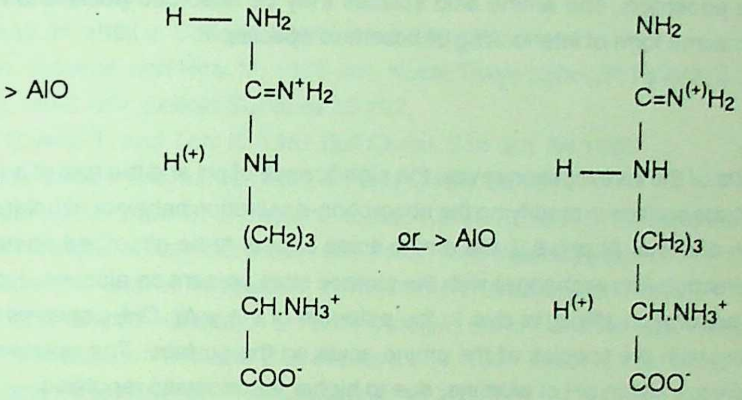
Type2
(proton or electron donor)



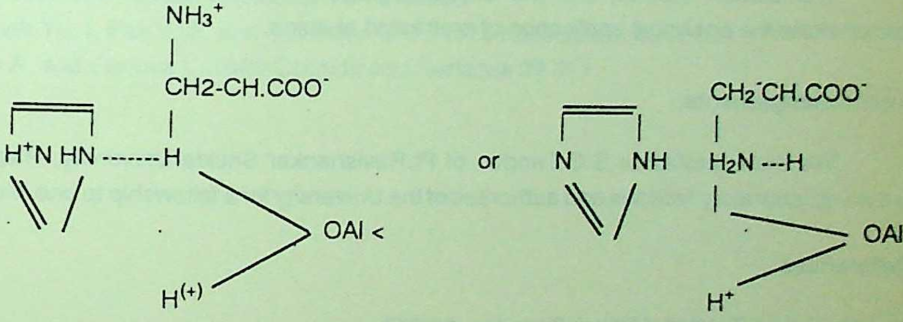
Type3
(electron donor)

Type 1 and 3 surface- groups should be the strongest hydrogen bond formers, while the type 2 group should be relatively weak, althouth one cannot rule out the possibility of this group being active outright. The amino acids used in this study have in their structure - NH_2 and > NH groups capable of hydrogen bond formation. So, it is likely that the forces operating in the process of adsorption is hydrogen bond formation, under the conditions where electrostatic repulsion si likely to be present, as shown below:

(a) For arginine,



(b) For histidine



This is also indicated by moderately fast nature of adsorption and is almost athermic nature.

The characterristic feature of this study is retardation of adsorption in presence of electrolytes in the adsorption bath (Fig.3), and significant desorption of the adsorbed species by the electrolytes (Fig.4). These observations may tempt one to say that ion-exchange, i.e., electrostatic forces may be responsible for the process, whose possibillty at pH<8.0, does not appear to be feasible (as explained earlier).

However, the role of electrolytes is manifold. The weak electrical forces responsible for accumulation of the adsorbed species on alumina, appear to be disturbed by the desorbing ions. These charged species, i.e. the desorbing ions may interact with the positively charged surface of the substrates, displacing the amino acid speies. Hydrogen bonding after all is a type of electrical force holding the species on the substrate. The anions present in the adsorption bath are also likely to weaken the hdrogen bonds, in their competition to reach the active sites present on the surface.

The S-type isotherms have been generally observed for the amino acids(Figs.1 &2),which may be due to;(i) large negative contribution of the solvent or a second solute and (ii) co- operative adosption. The amino acid species may be adsorbed packed in rows or clusters, through some form of interlocking of adsorbed species.

5. Conclusion

The results of the investigation reveal the significance of pH and the role of adsorbed ions on the substrate surface in modifying the absorption-desorption behavior of polar solutes (amino acids) on alumina. At pH>8.0, the amino acids appear to be adsorbed primarily via electrostatic interaction (ion exchange) with the surface sites present on alumina. However, at pH ≤ 8.0 the adsorption affinity is due to the potential of the > $AL.OH_{2+}$ species to from hydrogen bonding with the species of the amino acids on the surface. The uptacke of the amino acids decrease within pH of alumina, due to higher electrostatic repulsion.

The elution studies and the chromatographic seprations achived here amply demonstrate the analytical application of pretreated alumina.

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A study on the Pegmatites of Tongpal Area, Bastar District, M.P.

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Abstract. The pegmatites of Tongpal area, are mosly barren and highly weathered. Four zones have been recognized. Vein bodies commonly have discordant relationship and show pinch and swell structures. The contact between pegmatite and host rock is more or less sharp. Deformational effects have been recorded mainly in the quartz, felspar and muscovite. Myrmekite is rare but vein and patch type of perthites are common. Wall zone of pegmatites comprises of common minerals with some muscovite and tourmaline; lepidolite, muscovite and cassiterite are main constituents of intermediate zone. The present study reveals that these pegmatites are of magmatic origin.

Key words Pegmatite; Genesis.

1. Introduction

The investigated area covers the south eastern part of Bastar District, M.P. lying between $18^{\circ} 48'$ N to $18^{\circ} 45'$ N and $81^{\circ}47' 30''$ E to $81^{\circ}50'30''$ E and included in the Survey of India topographic sheet No. 64F/14. Tongpal village is situated in the southern part of Jagdalpur, on the Konta-Jagdalpur state highway. The pegmatites and associated metabasic rocks occur within the schists and gneisses. The Tongpal and the surrounding area has been mapped by Crookshank in 1938. Other workers who have studied the area are Fermor (1940), Ghose (1941), Deshpande (1976,81), Mukherjee, et al. (1979), Singh (1980,85), Mishra, et al. (1988), but no detaild literature is available regarding the genesis of pegmatites.

2. Regional Geology

The central part of the Indian Peninsular shield formed during the precambrian period, covering an area of 40,000 sq.kms.; it is bounded by Chhattisgarh basin in the north, Khondalite- Charnockite complex in the east and gneissic complex in the south and west. Regionally, the lithostratigraphic sequence of Bastar has been given by Crookshank in 1963. The revised stratigraphic classification of Bastar is given by Mishra et al. (1988) as below;

	Soil and alluvium
Intrusive Group	Quartz vein
	Pegmatite, Dolerite
Bengpal Group	Acid igneous rocks- grantie, migmatite.
	Metavolcanics - Metagabbro, Pyroxene granulite, Amphibolite
	Ferruginous rocks - Grunerite schist

Arenaceous rocks - Micaceous quartzites
Pelitic rocks - Micaschist
Calcareous rocks - Calc granulite

Basement undifferentiated

3. Geological Setting of the Area

The Tongpal hill is located a few hundred meters north of Tongpal village extending from Buddapada village in east to Tanakvada village in the north covering about 5.7 kmg. length. The area under study is mainly comprised of Bengpal group of rocks and intrusive pegmatites. Pegmatites are equivalent to middle Proterozoic age and trending NNW-SSE direction with low dip. It occurs as lenticular veins and zoned bodies and is exposed throughout the area. The main litho-units include:

Quartz vein
Dolerite
Pegmatite
Amphibolite
Pyroxene granulite
Mica Schist
Micaceous quartzite (Muscovite < Biotite)

The pegmatites show sharp contact with instrusive nature with amphibolites with and pyroxene granulites of the present area. Apart from large bodies a number of small offshoots of pegmatites trend in various directions. Concordant as well as discordant relationships between pegmatites and metabasics have been exhibited. Emplacement of pegmatites in the various rock types took place in two generations. Barren types occur in the north-east part of Tongpal village. Pegmatites of Tankavada area contain some amount of mica flakes. Pegmatites occurring on the left flank of baru river, near Marenga village show discrete zoning. Pinch and swell structures are more prominent in the lenticular bodies. Isolated lensoid pegmatites are also reported in S-W part of Tongpal. Zoned pegmatites show sharp contacts with country rocks. Four zones have been recognised- border, wall, intermediate and core. The Border zone of pegmatite body (maximum thickness is 0.5 meter) mainly contains quartz, felspar and muscovite. The size of grains is smaller in respect to other inner zones. The wall zone is conspicuously developed and it has a mineral assemblage of coarse grained quartz, microcline, perthite, plagioclase with little amount of muscovite and biotite. The width of this zone ranges from 0.5 to 2 meters. The intermediate zone is developed irregularly and the thickness varies from 0.3 to 3.3 meters. This zone contains quartz, microcline, perthite, albite, oligoclase with little amount of muscovite, tourmaline, lepidolite, cassiterite and apatite. The innermost part of pegmatite body, forming the 'Core zone', comprises of large highly fractured quartz crystals in the form of pods. Simple and unzoned pegmatites are more weathered than the zoned pegmatites.

4. Petrography

The pegmatite is medium to coarse grained, light coloured, composed of quartz and felspars with some amount of biotite and muscovite. Graphic intergrowth between quartz and alkalifelspar is clearly visible in hand specimens. Microscopically, the simple pegmatites show much more diformational effects than the zoned type. The minerals present in order of abundance are quartz, potash felspar, plagioclase, muscovite, and biotite with accessory minerals tourmaline, apatite, lepidolite, magnetite and cassiterite etc. Smocky quartz is rare. Quartz grains are of two generations. Large anhedral form is common but this also occurs as veinlets. It shows deformational effects such as undulating extinction and fractures. These grains are highly crushed and have corroded margin. Quartz grains of second generation are subhedral, inclusionless and shows simple extinction.

Potash felspar (microcline and microcline perthite) occurs as elliptical and subhedral form. Micracline is common and shows intergrowth relation with oligoclase. Perthites of string, vein and patch type seem to have originated by exsolution. Deformational effects are recorded in microcline perthite as fractures and displacement in twin lamellaе (Fig.1) Quartz and small flakes of mica are the common inclusions Plagioclase (oligoclase) occurs as large crystals and exhibits well developed polysynthetic twinning. It shows myrmekitic textures. Crystals of plagioclase are fractured displaced and bent twin lamellae are also present. (Fig.2)

Muscovite is more abundant than biotite and its flakes are diversely oriented. The relatively coarser flakes are almost free from inclusion. Biotite flakes also occur alogwith the flakes of muscovite, indicating simultaneous growth. Deformational effects are also recorded by the presence of deformed and bent cleavage plane with twisted flakes. Uneven edges of muscovite flakes indicate dissolution. Biotite is present only in weatheredpegmatites. It occurs as small flacks with bird's eye maple surface showing twisting phemomenon.

Accessory minerals like columanar crystals of tourmaline occur as clustres in felspars. It Shows variable black to yellowish white colour with parallel extinction. Lepidolite appears as aggregates of tiny scales, showing brilliant second order pink and green polarization colour. Yellowish green apatite in subhedral to anhedral form is present as inclusions in quartz and felspar. Cassiterite and magnetite occur as tiny anhedral crystals.

5. Origin of Pegmatites

The pegmatite occurs as intrusives in the form of veins, lenses and zoned oval shaped bodies. The fractional crystallization of the pegmatite body proceeds after the formation of fine- grained border zone. The grain size of various minerals gradually increases towards inner zone because of the slow cooling rate of crystallization of the melt. The slow rateof cooling was possible owing to the presence of the chilled (fine-grained) border zone which restricted the loss of heat and volatiles. Presence of lepidolite, fluorite and cassiterite

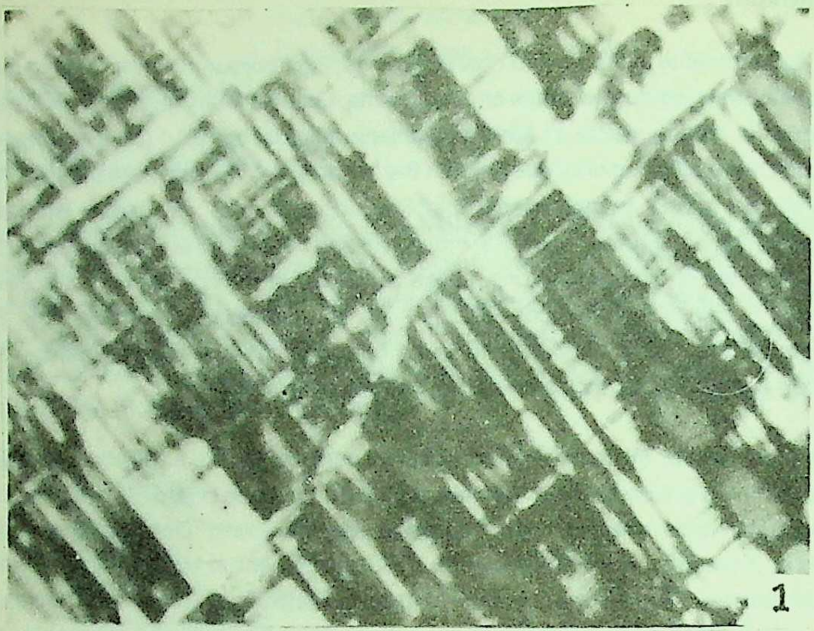

Fig.1: Photomicrograph of microcline perthite showing displacement of twin lamellae (C.N.Enlarged).

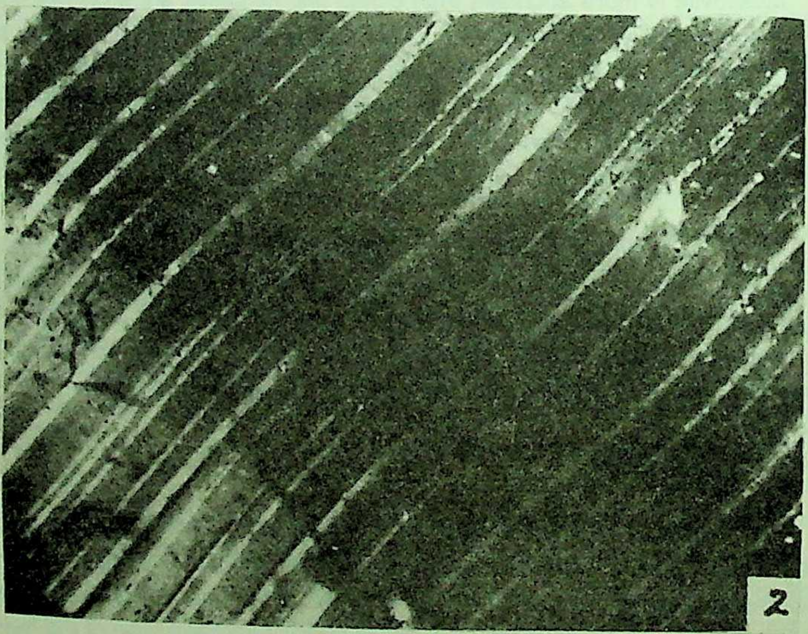

Fig.2: Photomicrograph of oligoclase showing deformed twin band is pegmatite vein (C.N. Enlarged).

indicate the effect of pneumatolysis (Deshpande et al. 1981). Mullick, Sahu and Panchapakesan (1982) studied the inclusions of beryl from pegmatites of Bastar, M.P. and concluded that there were an immiscible aqueous phase and a silicate melt at the time of pegmatite empacement in pneumatolytic stage. Although Basu (1985) advocated several phases of tin mineralization of the area in the metasomatically altered variety of greisenised ande albitized pegmatites.

Mussovite is the first mineral to crystallise from pegmatitic melt, with little biotite. Both are present as inculusions in the felspar. All the micas are hapazrdly oriented. Afterwards, subhedral apatite has been developed. It is present in quartz and feldspar, indicating that the former mineral is older than the later ones. Long crystals of tourmaline with lepidolite, crytallized after the formation of apatite. The former mineral occurs as well diveloped crystals in felspars. The preponderance of potash felspar over plagioclase, can be explained that the available alkali in the system has been utilized in the formation of microcline and perthite under poor hydrous phase. Muscovite poor and potash felspar rich nature of pegmatites also supports the crystallization almost in absence fohydrous phase. The presence of cassiterite, columbite-tantalite and beryl indicates the minor influence of pneumatolyses on pegmatite at later stage.

The crushed nature, presence of pinch -and- swell structure, sharp and distinct contacts with host rocks, fractured cores of zoned pegmatites, absence of zoning in plagioclase, presence of porphyritic, graphic & myrmekitic texture, perthites of exsolution,fraction of crystals in zoned pegmatite, all these field and petrological evidences strongly support the magmatic origin of these pegmatites.

Acknowledgements

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Formation of Biotite Selvages in Granitic Rocks, District Gaya, Bihar.

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Abstract. Precambrian coeval basic rocks metamorphosed equivalent to low grade amphibolite facies, have been granized along their fractured zones. Biotite selvages/sur-micaceous enclaves are formed as a result of the advent of potash rich solutions with the continuation of elevated temperature. Sufficient supply of metasomatising material and fractured zones have facilitated the complete transformation of the metabasic rocks into granodiorite, leaving a biotite rich zones in the Bramhajoni Hill,Gaya,Bihar.

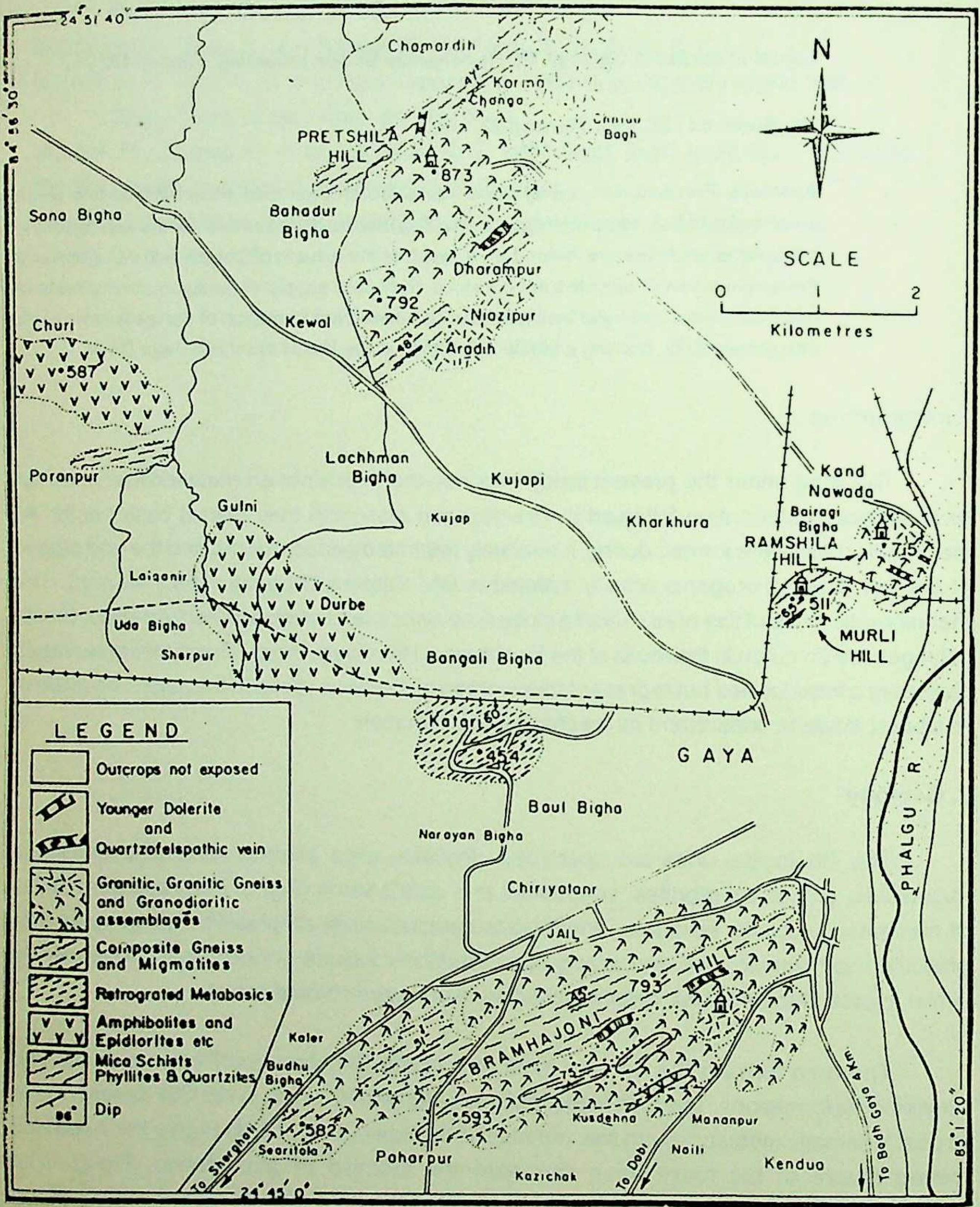
1.Introduction

The area under the present study exposes the Precambrian metasediments which have suffered deformation followed by granitization alongwith their coeval basic rocks. All the granitic rocks were formed during a relatively restricted period that marked the end stages of a major pulse of orogenic activity initiated in late Satpura Orogeny (950 ± 40 my.). The metasomatic rocks of this area exhibit a close time-space relationship to fracture zone. Biotite selvages are common in the rocks of the Bramhajoni Hill, Gaya, Bihar. These biotite selvages represent a transformed but representative portion of the basic rocks.In this paper an attempt has been made to understand its mechanism of formation.

2. Geology

The lithological units are quartzites, phyllites, mica schists, composite gneisses, migmatites, gneissose granites, pegmatites and quartz veins (Fig.1). Prograde derivatives of metabasics include epidiorite, amphibolite,epidote-zoisite amphibolite, plagioclass amphibolite, hornblendic rock and retrograde derivatives include chlorite-biotite- hornblende schist, biotite-chlorite schist, chlorite schist and kaolinized chlorite schist.

The area forming the northen most part of the ChhotanagpurRegion exhibits two phases of deformations. The fault plane is aligned along ENE-WSW to NE-SW direction. Syn to post kinematic metasomatism has yielded hybrid rocks (Kuity, 1978) Highly fractured and jointed nature of the rocks have facilitated the process of granitization (Fig.2). The metasomatism was not equally affective throughout the area may be due to supply of the material, nonavailability of the suitable locales and variable response to transformation (Fig.3).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Niazipur
Churi
Kewal
Aradih
48
587
Paranpur
Lachhman Bigha
Kujapi
Kujap
Kharkhura
Kand Nawada
Bairagi Bigha
RAMSHILA HILL
715
511
63
MURLI HILL
Gulni
Laiganir
Uda Bigha
Durbe
Sherpur
Bangali Bigha
Katari
60
454
GAYA
Baul Bigha
Narayan Bigha
PHALGU R.
Chiriyatanr
JAIL
793
HILL
BRAMHAJONI
Kaler
Budhu Bigha
75
593
582
Searitola
Paharpur
Kusdehna
Mananpur
Naili
Kenduа
Kazichak
To Sherghati
To Dobi
To Bodh Gaya 4 Km.
24°45'0"
85°1'20"
LEGEND
Outcrops not exposed
Younger Dolerite and Quartzofelspathic vein
Granitic, Granitic Gneiss and Granodioritic assemblages
Composite Gneiss and Migmatites
Retrograted Metabasics
Amphibolites and Epidiorites etc
Mica Schists Phyllites & Quartzites
Dip
86°

Fig. 1: Geological map of the area around Gaya, Bihar.

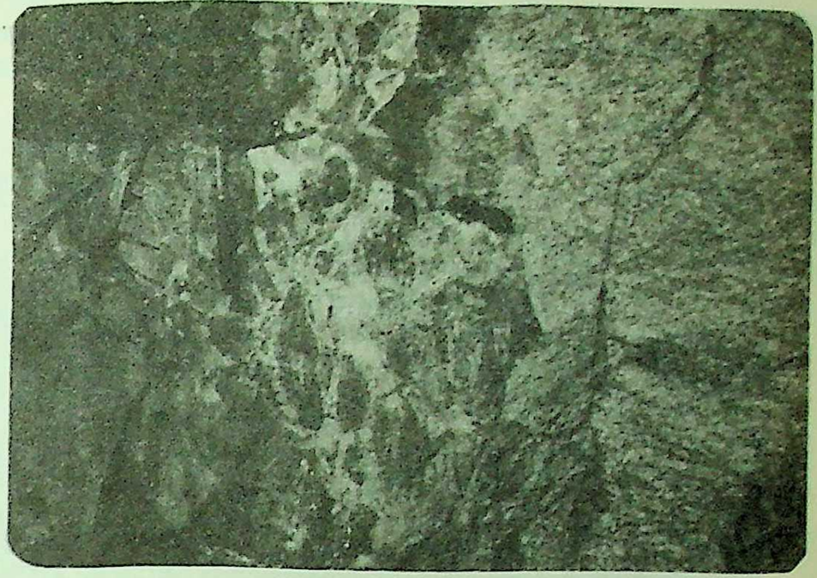

Fig. 2: Feldspathization of metabasic rocks. Gaya

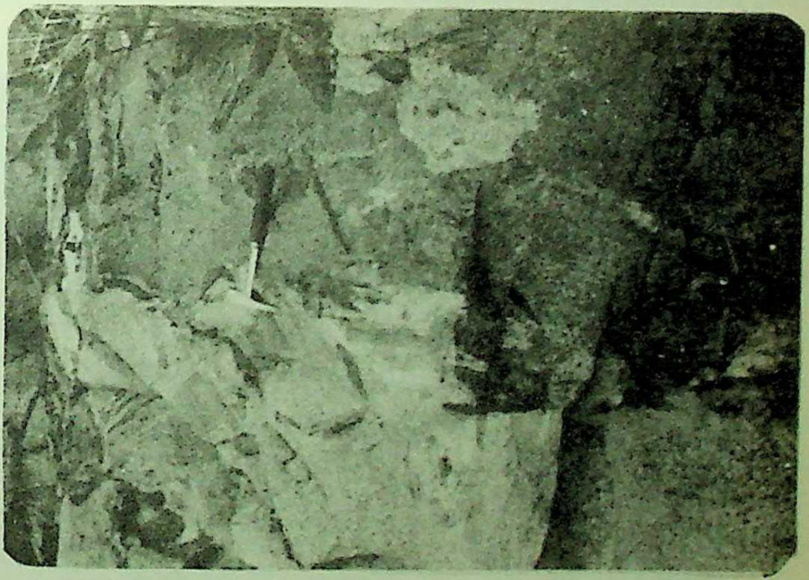

Fig. 3: Metabasic rocks relatively less affected by feldspathization, Gaya

3. Petrography

The biotite selvages are dark coloured and show poorly defined foliation (Fig.4). The biotite flakes are clearly visible in hand specimen and the maximunm size recorded is 1cmX0.75 cm (Fig.5). Under the microscope, the rock shows a schistose character. Clustres of deep brown pleochroic biotite is the main constituent. Plagioclase crystals studed with

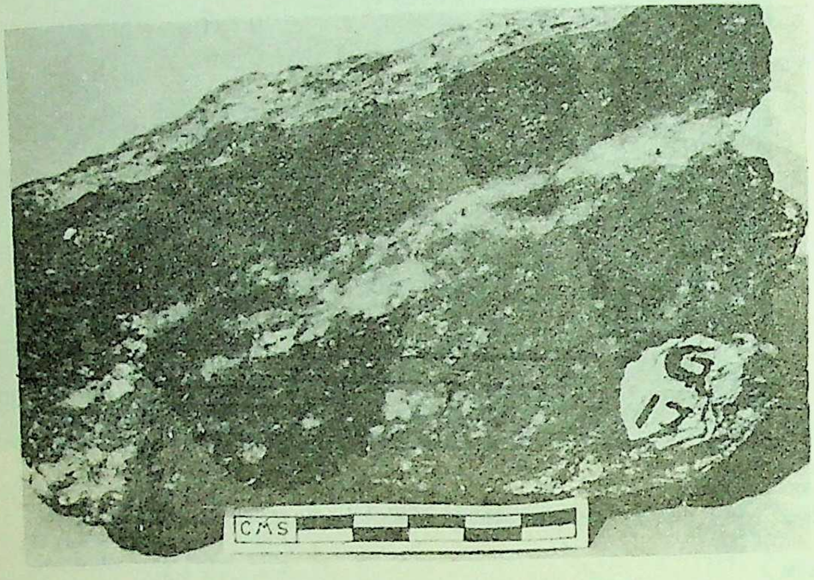

Fig. 4: Megascopic aspects of a biotite selvage/surmicaceous enclave from the granodioritic rocks of Gaya.

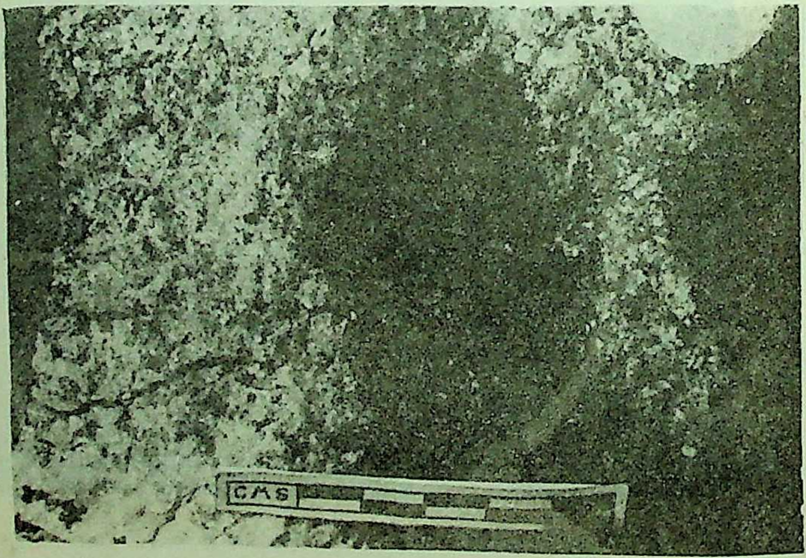

Fig. 5 Hand Specimen photograph of a biotite selvage from zone of feldspathization, Gaya

quartz inclusions and biotite flakes often lying across it. Both fresh and altered plagioclase are present Potash feldspar grains are small, fresh and show simple twinning . Zoned plagioclase with core of andesine to oligoclase is surrounded by a potassic feldspar. Quartz occurs as elongated. strained grains with irregular margins and numerous inclusions. Muscovite is mostly associated with biotite. Biotite is greenish brown and its alteration to chlorite is mostly partial. In few cases occurrence of very small biotite flakes are noted in the muscovite. The common secondary mineral includes chlorite and epidote.

The feldspar content and coloration indices have been calculated (Table-1) from the modal analyses of biotite selvages, and are plotted in Fig.6 as proposed by Jung and Brousse (1959).Different types of microgranular enclaves occupy distinct field in the figure. The biotite selvages fall in the field of surmicaceous enclaves.

Table 1: Modal Compasition for surmicaceous enclaves from the Aigoual, St. Guiral, Liron and Gaya.

Mineral	Area	Aigoual	St.Guiral	Liron	Gaya
Biotite		66	66	60	60
Quartz		7.5	0.5	2.5	10
K-feldspar		2	8	2.5	6
Plagioclase		23	9	17	10
Others		1.5	16	18	14

* Average of 5 samles.

Indices of feldspatic content and coloration

Indices	Area	Aigoual	St.Guiral	Liron	Gaya
Felds. Content		8	47	13	37
Coloration		67.5	82.5	78	64

4. Chemstry of biotite

The biotite has been separated from the biotite selvages, with the sieve +80 mesh and finally by hand picking with the aid of a binocular microscope. These have been powdered by adding alcohol and analysed after Shapiro(1975). The chemical analyses (Table-2) have been plotted on the triangular diagram FeO + MnO - Fe_2O_3 + TiO_2 - MgO (Fig.7) and the point falls in the granitic field (Engel and Engel, 1960).

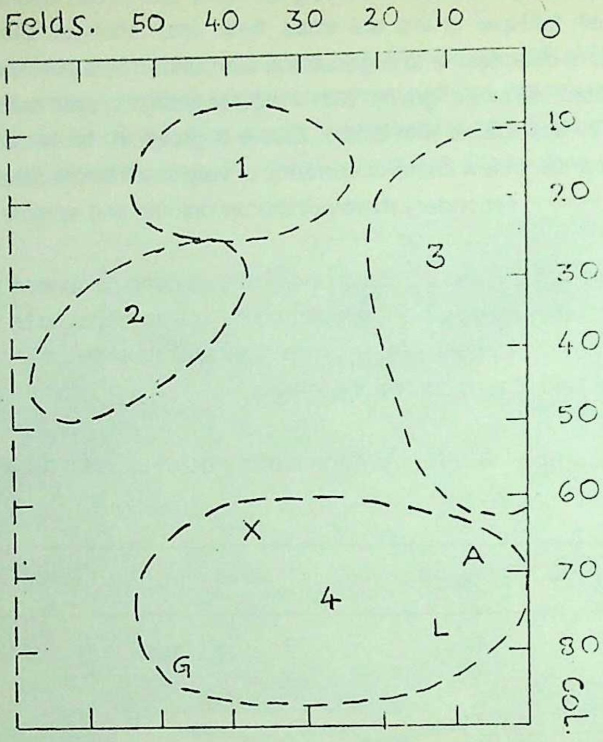

Fig. 6: Fields of different enclaves. 1-pale enclaves, 2-dark monzoitic enclaves, 3-dark plagioclase rich enclaves, 4-biotite selvages/surmicaceous enclaves, Locations: A-Aigoual G-Guiral L-Liron X-Gaya.

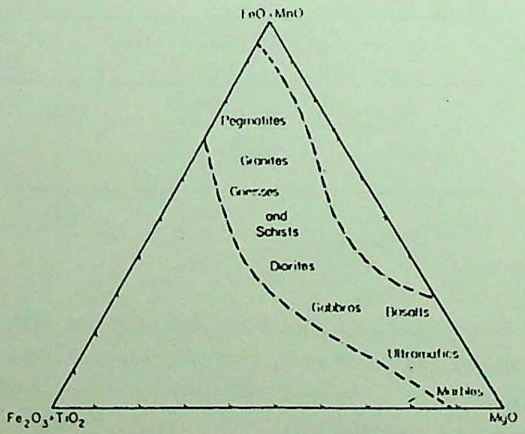

Fig. 7: Biotite composition of the microcular enclaves occuring in different rocks. X-Biotite from surmicaceous enclaves, Gaya.

Table 2: Chemical Analyses (Wt.%) of biotite from Biotite selvages,Gaya (Averge of 3 samples).

Oxides	SiO_2	Al_2O_3	Fe_2O_3	FeO	MgO	CaO	Na_2O	K_2O	TiO_2	MnO
Wt.%	35.25	19.31	3.30	18.41	7.92	0.88	0.27	8.01	2.39	0.18

5. Fromation of Biotite Selvages

Hughes (1970) concluded that the biotite selvages in migmatites originate at phase boundaries between rock with a hydrothermal porefluid and rock with a silicate porefluid. Deb and Kuity (1983) stated that the granitophyle components activated by vapour, volatile and minor liquid (?) have moved to accomplish the transformation reaction of amphibolites. Nockolds (1934) described the discontinuous reaction series of the basic minerals which ends with the formation of biotite. The series is as follows-

Monoclinic pyroxene ——— brown Hb ——— green Hb ——— biotite
" " ——— ortho Py ——— biotite

Reynolds(1946) examplified 'replacement' bodies of Newry granodiorite rimmed by biotite enriched hornfels zones. The biotite rich zone was earlier considered as Mg-Fe front and later on the some has been considered as a real basic front. The Deoro granite pluton in Ontario has caught up patches of old basic rocks and is characterised by a development of biotite and microcline (saha,1959).

Karkare and Srivastava (1985) suggeated that the Sukma granitoids (Bastar) are the products of palingenesis of preexisting rocks formed under the influence of metasomatism. Norton (1988) stated that in magmatic environments advective transport of mass and energy is the primary cause of matasomatic alteration which persists for long times and imposes systematic chemical and mineral alteration patterns. Marre (1986) recorded the development of augen structure and aligned plagioclases moulded by stringar of biotite, in the margin of a pluton. The foliation becomes inereasingly intense and parallel to the country rock contact. Absence of zonation,synneusis and oriented segregation of crystals supports the metasomatic origin of these rocks. Intimately associated metabasic rocks of the area (Gaya) have also been compailed to respond the transforming processes accordingly. A biotite rich zone produced as a result of partial assimilation, has been observed around amphibolites of Kanker (Lakshmi Narayan and Kuity,1988).

6. Discussion and conclusion

Biotite selvages of Gaya may also be called as surmicaceous enclaves as they contain more than 50% biotite by volume as proposed by Didier(1973). The chemical composition of biotite selvages is similar to that of biotite in granite. These selvages are sourrounded by a rock dominated by feldspar (60 to 80 mode %) and the other minerals are mainly quartz and biotite. Biotite occurs as dispersed manner in the relatively more transformed rocks where as in the least altered rocks it occurs as parallel to the enclaves. SiO_2 Vs $Na_2O + K_2O$ (Wt%) of unaltered metabasic rocks have been plotted (Fig.8; Table- 3). The plots indicate that the

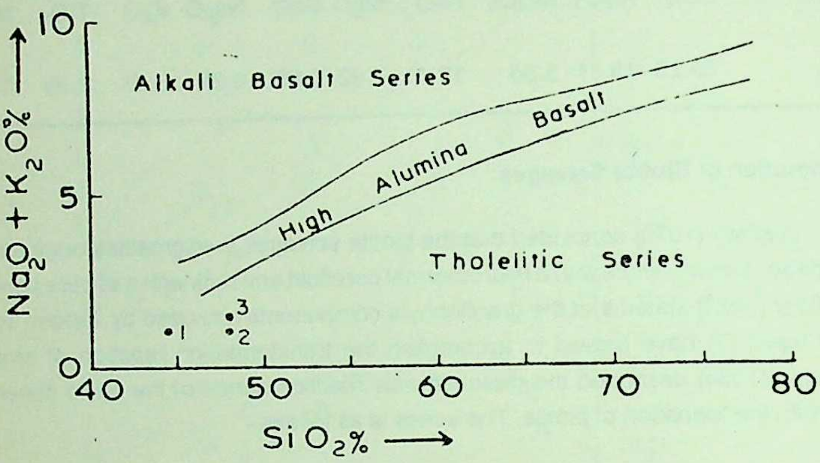

Fig. 8: Diagram showing variation of SiO_2 vs $Na_2O + K_2O$ (wt %) for the basaltic rock series. Plots indicate least altered metabasics of Gaya (After Kuno, 1966).

parent material of these rocks was a comparatively low SiO_2 and low alkali tholeiitic basalt (Kuno, 1966). Harme (1962) is in the openion that the 'biotite crust' represent the mobilisates that were not expelled.Mehnert (1968) considered the presence of drop like quartz and obliquely occurring biotite in plagioclase as the positive sign of recrystallisation. He also suggested the formation of mobilisates during the transformation into biotite. Solutions loded

Table 3: Chemical Analyses (Wt%) of Metabasic Rocks of Gaya, Bihar.

Oxides	Sample Nos.	1	2	3
SiO_2		46.12	49.09	49.74
TiO_2		2.7	2.35	1.44
Al_2O_3		13.62	14.55	14.59
Fe_2O_3		4.05	4.82	4.76
FeO		9.58	8.59	8.41
MnO		0.31	0.20	0.26
MgO		6.35	5.51	5.38
CaO		15.65	13.42	13.72
Na_2O		1.14	0.91	1.02
K2O		0.19	0.32	0.42
P_2O_5		0.18	0.22	0.22

with granitophyle elements under elevated temperateur transformed hornblende into biotite. The rocks undergoing metamorphism lose potassium and silica, which migrate towards the place of lower free energy (fracture zone) causing granitization of the rocks in the area (Marmo,1971). Absence of hornblende in these enclaves may be accounted in two ways, either this mineral has not formed or this has changed completely to biotite under favourable potash rich condition.

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Radioelement distribution in Leptynites from Eastern Ghat Granulite Terrain

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Abstract. Leptynites are important rock type in the Visakhapatnam area and they are categorised into grey and cream varieties. At a glance. It is noticed from the data that the radioelement concentrations in grey leptynites are lower while they are higher in the cream leptynites. The concentrations of U, Th, K and Th/U ratios of grey ieptynites do not represent the primary distribution but indicate their expulsion in a hydrous partial melt from the destabilised mineral sites/ dislocations of k-feldspar, biotite and other accessory phases. The expelled U, Th, K and other LIL elements during granulite facies metamorphism were carried up in hydrous granitic partial melts along weak shear planes/zones and crystallized in surrounding cream leptynites. The enrichment of radioelements in the cream leptynites may be possible by two reasons: 1. Crystallization of radioelement bearing partial melts in the cream leptynites 2. The radioelements already resided in the accessory phases present in cream leptynites. The expulsion of U, Th and K from grey leptynites during granulite facies metamorphism is simultaneously followed by the crystallization of partial melts in the cream variety. The higher rediogenic heat generation (2.17) values of leptynites reveals that the Precambrian crust of the Eastern Ghats Granulite Terrain characterised by high radioactivity.

Key words: Leptynites- VisaKhapatnam- Radioelements- Metamorphism- partial melting- Radiogenic Reat Generation.

1. Introduction

The chemical signatures of insoluble and less fractionated elements (REE, U,Th, Sc) during sedimentary and metamorphic processes in the precambrian metasediments play a pivotal role in estimating the crustal composition and protoliths and in view of this the radioelement (U. Th. K) geochemistry has been studied by many workers in India and elsewhere (Wildman and Haskin 1973, Nance and Taylor 1977, Boak et al 1982 Mc lennan and Taylor 1983, 1984, 1985). The present investigation deals with the distribution of U, Th and K in leptynites of Visakhapatnam area (Latt. 17°40' and 17° 50' 30" N; Long. 83° 12' and 83° 20' E). which is a part of the Eastern Ghat Granulite Terrain comprising charnockites (tonalites & granodiorites). two pyroxene mafic granulites, leptynites, khondalites. calc-granulites and quartzites. The clac- granulite and quartzite bands are conformable and gradational with leptynites (Rao and Nageswara Rao 1987). Grey and cream leptynites have been recognised based on field relations, mineralogy and immobile element geochemistry (Nageswara Rao 1989) and these two varieties occur at different localities in the Visakhapatnam proper. Grey leptynites are migmatised (Fig.1) and partial melts have crystallized as cream leptynites which often show intrusive relationship with grey leptynites (Nageswara Rao 1989, Nageswara Rao and Rao 1990)

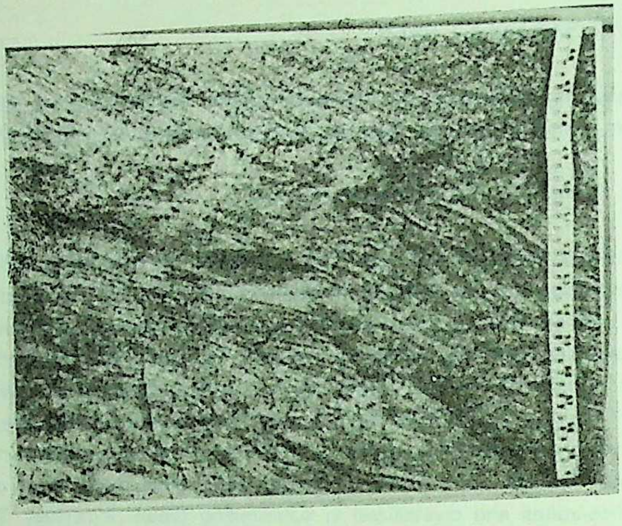

Fig. 1: Migmatitic layering in the grey leptynites.

Grey and cream leptynites rocks are essentially made up of quartz, plagioclase (Gl: vol. %;20.2 ; An : 8.5 CL. vo1. %: 17.8, An: 5.6). k-feldspar, garnet, biotite and the accessories are sillimanite (GL: significant amount of Fe^{3+}, vol. %: 2.5; CL: O.3%, spinel(GL: significant concentration of Fe^{2+} vol.%: o.9; CL: O.03%), zircon, apatite and monazite. Quartz, K-feldspar and biotite minerals are slightly more abundant in the cream leptynites in contrast to grey type.The leptynites of Visakhapatnam are considered to be the metamorphic (granulite facies) derivatives of detrital sediment (greywacke to clayey) plus rhyolite tuff admixtures (Nageswara Rao 1989) and this type of origin was substantiated by earlier workers (Losert 1971; Bhattacharya 1973, Pitchimuthu 1982. Stankova 1983. Srikantappa et al 1985).

2. Analytical Methods

Uranium, thorium and Potassium elements for ten samples have been determined by Gamma Ray Spectrometry at AMD, Hyderabad. U and Th elements of remaining ten samples were estimated by Neutron Activation Analysis at BARC, Bombay and K was analysed by X-Ray Fluoroscence Spectronetry at NGRI, Hyderabad. The details of instrumentation used to determine the elements is described in the Table1.

3. Results and discussions

Table 1 shows the U, Th, K Th/U and Heat Generation Units (HGU) on grey and cream leptynites. At a glance, it is observed from the data that the radioelements. Th/U and HGU

Table 1: Radioelement content (in ppm), Th/U ratios and HGU values in the leptynites of Visakhapatnam area

Sample Nos.	U	Th	K (X10)	Th/U	HGU= H W/m^3
GL7[a]	2.40	2.1	3.05[b]	0.875	1.04
GL11[c]	0.80	2.0	3.10[c]	2.500	0.63
GL12[c]	1.00	2.0	3.80[c]	1.000	0.74
GL23[c]	1.00	1.0	2.70[b]	1.000	0.57
GL24[a]	1.10	0.5	3.38[b]	0.454	0.64
GL34[a]	5.00	0.9	2.44[b]	0.225	1.56
GL58[a]	6.20	1.0	3.62[b]	0.161	2.00
GL73[a]	1.00	1.0	3.70[c]	1.000	0.66
GL88[c]	1.00	0.7	3.10[c]	0.700	0.58
GL122[a]	1.00	0.5	2.29[b]	0.500	0.72
Gl127[a]	3.80	19.3	2.16[b]	5.078	2.51
Ave.of GL	2.20	2.8	3.03	1.226	1.05
CL21[c]	10.30	9.4	4.89[c]	0.912	3.75
CL62[a]	4.00	19.0	4.40[b]	4.750	2.75
CL72[c]	1.00	2.0	3.40[c]	2.000	0.70
CL86[a]	5.60	101.0	2.96[b]	18.035	8.71
CL90[a]	6.40	79.0	3.86[b]	12.343	7.49
CL98[c]	1.00	11.0	3.40[c]	11.000	1.33
CL109[a]	7.00	31.6	3.77[b]	4.514	4.35
CL125[c]	1.00	6.0	3.60[b]	6.000	1.00
CL135[c]	1.00	18.0	3.20[c]	18.000	1.79
Ave.of CL	3.70	30.7	3.72	8.617	3.54
Mean of GL+CL	3.05	15.4	3.34	4.552	2.17

a= U,Th, K were determined by Neutron Activation Analysis (Procedures adopted by Parthasaradhy et al 1985)

b= K was analysed by γ-Ray Fluorscence Spectrometry

c=U,Th,K were estimated by X-Ray Spectrometry

d= The errors due to counting statistics is less than ± 5%in both NAA and rRs methods

GL= Grey Leptynites;CL= Cream Leptynites values are lower in grey variety and higher in cream leptynites (Table 1). The mean contents of U (3.08), Th (15.4), K (3.34), Th/U (4.552) and HGU (2.17) for leptynites as a whole (GL+CL) compares well with the values of Post- Archean upper continental crust and Post Archeean sediments (Table2).

Table 2: Radioelement content (in ppm) of Post-Archaean lithologies compared to present study

S.No.	Litho-unit	U	Th	K (X10)	Th/U	THU= $\mu W/m^3$
1.	Upper continental crust (Taylor 1985)	2.80	10.70	2.80	3.80	1.72
2.	Upper continental crust (Udo Haack 1983)	3.10	12.40	2.34	4.00	1.87
3.	Mature clastic sediment (Taylor 1985)	3.20	14.00	-	4.80	-
4.	Shale (Mc Lennan & Taylor 1980)	3.10	14.60	-	4.70	-
5.	Leptynites (20)	3.08	15.40	3.34	4.55	2.17
	a. Grey Leptynites (11)	2.20	2.81	3.03	1.22	1.05
	b. Cream Leptynites (9)	4.12	29.60	3.71	6.85	3.41

The significant contribution of U and Th in the Archean high grade metasediments of Australia (Taylor et al 1986) and their marked variations in the Eastern Ghat Khondalites may be attributed to the nature of the provenance and abundance of metamorphic and detrital minerals which are concentrated during sedimentary processes (Divakara Rao 1982). In the Archeon metasediments the U, Th, K and other LIL elements are depleted during high pressure granulite metamorphism (Rollinson and Windley 1980, Okeke et al 1983 Taylor et al 1986). From the earlier studies, it is evident that the degree of depletion depends on position of U and Th in the protoliths, the fluid phase and the stabilities of various accessory phases during granulite facies metamorphism (Dostal and Capedri 1978, Rudnick et al 1983.) According to Constable and Hubbard (1981) the Th in granite gneisses may lie in the dislocations of k-feldspar and biotite whilst a large proportion of U is retained in more refractory minerals. Radioactive and other LIL elements are strongly enriched in the upper crust (Udo Haack 1983) and these insoluble elements are quantitatively transferred into terrigenous sedimentary rocks (Mc Lennan et al 1980, Taylor et al 1986).

The leptynites from Visakhapatnam area are characterised by variable distribution of radioelements (Table1).The average radioelement content of grey leptynites are lower (U=2.20. Th= 2.81,K=3.03 Th/U=1.226) and these values cannot indicate the primary distribution because the protoliths (upper crustal sediments) of grey leptynite have higher radioelement concentrations. A more probable explanation for U, Th and K gepletion inthe grey leptynites could be the preocesses of their formation by granulite facies metamorphism where the mineral sites of k- feldspar. biotite and other accessory phases (apatite, monazite, zircon) were estabilised, resulting in the expulsion of U, Th and other LIL elements in a hydrous partial melt from the destabilised mineral sites/ dislocations.Many petrologists have found that granulite facies rocks are depleted in K, Rb, Th and U contents, conceivably because of their preferential incorporation into hydrous silicate partial melts or hydrous fluids that migrated out of the bodies (Tarney and Windley1977).

The expelled elements were carried up in hydrous granitic partial melts along weak shear planes/ zones and crystallization in the neighbouring cream leptynites (Nageswara Rao 1989). As a result of this the radioelement contents are higher in the cream leptynites (U-3.70.Th- 30.71. K = 3.72 , Th/U= 8.617). The enrichment of radioelements in the cream leptynites may be possible due to two reasons: 1. The crystallization of radioelement bearing granitic partial melts in the cream leptynites 2. The radioelements already resided in the accessory phases of cream variety. The expulsion of radioactive elements in a hydrous granitic partial melt from the grey leptynites by metamorphism is simultaneously followed by the crystallization of these partial melt in the cream leptynites ingranulite facies conditions around 2,600/2,500 M.a.

4. Radiogenic Heat Generation .

Heat generation data (HGU) for leptynites are calculated from the radioactive elements and they were converted into radiogenic heat (Birch 1954, 1965) There are marked variations in heat generation (THU- μ w/m^3) values of leptynite samples as a whole (Table1). but the average HGU of cream leptynites (3.54) is more in contrast to the grey leptynite (1.05) ascribed due to enrichment of radioelements. The mean HGU value of Visakhapatnam leptynites (2.17) is high when compared to the values of upper continental crust (Table2). greywacke from chitradrug belt (0.62) and metapelite from Holenarsipur belt (1.08) from Karnataka (Gupta et al 1989) suggests that the Precambrian crust of Eastern Ghat Granulite Terrain was characterised by higher radioactivity.

5. Conclusions

The mean U, Th, K and Th/U ratios of leptynites (GL+CL) from Vishkhapatnam can be fairly compared with post-Archaean upper crustal and post- Archaean sediment values. The distribution of radioelements suggests that they have been depleted in the grey leptynites during the granulite facies metamorphism. However, it is observed that these depleted elements restored in the cream leptynites by the crystallization of granitic partial melts in the cream leptynites. The radiogenic heat production values of leptynitic rocks appears to be higher than in the rocks of upper continental crust and post- Archean sedimentsreveals that the Precambrian crust of Eastern Ghats Granulite Terrain was characterised by higher radioactivity.

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Genesis and behaviour of uranium in Pitepani basic Metavolcanics group around Bodal, Bhandaritola, District Rajnandgaon, M.P.

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Abstract. Radioactive basic metavolcanics i.e. amphibolites of Bodal-Bhandaritola areas have been assigned Lower Proterozoic age of Nandgaon Group. Meso to epizonal reconstitution of acidic metavolcanics as rhyolitic suites during polyphase tectonism with accompanying metamorphism, shearing and hydrothermal activity has resulted in the formation of ore grade amphibolite bodies. Uranium mineralisation is essentially restricted to shears trending N 18° E-S 18° W and N 40° W-S 40° E in Bhandaritola and Bodal respectively which transgress from amphibolite in the east to quartz porphyry in west. Peribatholithic contact of Dongargarh granite with the uraniferous amphibolite and quartz porphyry shows distinct effects of protoclasis aiding in enrichment of uranium (upto 8400 ppm) within host rocks. Notably an increase in uranium content shows an increase in Si, A1, total alkalies and Pb, but Mg, Ca, Fe, Mn, P_2O_5 show a concomitant decrease, while the trace elements (Ba, Rb, Sr, Ni, Zn and Cu) show an inconsistent and eratic behaviour. It has been suggested that amphinbolites with higher Si, Al, total alkalies and Pb will be uraniferous, even though surface manifestation of uranium is poor or even lacking.

Key words - Pitepani basic metavolcanics, Bodal Bhandaritola, Uranium, Amphibolite, Nandgaon Group.

1. Introduction

Bodal (20°39'N, 80°45' 40" E; 64 D/14) and Bhandaritola (20° 42' N, 80° 41' 30" E ; 64D/10) areas in Rajnandgaon district of Central India form a part of the volcano-plutonic complex correlatable with the "Precambrian IV" of Dongargaon system (Sarkar 1957, 1958, 1983; Table 1). The major chronostratigraphic units are metamorphic equivalents of Pitepani volcanics (amphibolites), Bijli Rhyolite (Quartz porphyry/silicious rocks), Sakoli group (Shale, quartzite), gabbro, dolerite and Dongargarh granites. Peribatholithic contact of Dongargarh granite is in the west of amphibolite and quartz porphyry of the Bhandaritola area. Sakoli shales are exposed in association with the brecciated quartzite and tuffs to the west of Badal-Bhandaritola areas. Presence of rhyolite and quartz porphyry X enolilths within the amphibolites and shale in rhyolite and quartz porphyry xenoliths within the amphibolites and shale in rhyolite and quartz porphyry characterises the stratigraphy of the terrain (Fig.1)

Regionally, the formations trend in N 18° E-S 18° W with steep dips (70° - 80°) due east in Bhandaritola while N 40° W-S 40°E with steep dips (60° - 70°) due east in Bodal. The

Table 1: The Regional Stratigraphic Succession of Bodal + Bhandaritola Area, Distt. Rajnangaon, M.P.

	Dongargarh belt (after sarkar et al. 1957 & Sarkar 1983)		Bodal- Bhandaritola sequence
570 M A Upper Proterozoic	Chattisgarh Super Group	Raipur Group Chandrapur Group	Brecciated ferruginous quartz veins.
900 Ma		Unconfirmity	
		Khairgarh Orogenic phase (c.900 MA ?)	Dolerite dykes very fresh, unaltered.
— Middle Proterozoic	Khairagarh Group Dongargarh Super Group	Mangikhuta Volcanics Karutola sandstone Sitagota volcanis (1443 Ma) Intertrappean shale (1760 Ma) Bortalao sandstones basal shale (1610 Ma)	Gabbroic bodies/ diabase
1600 MA		Unconformity	
— Lower Proterozoic	Nandgaon Group Dongargath Super Group	Dongargarh Granite (c.2200 MA) Pitepani volcanics Bijli Rhyolites (c. 2200 MA)	Granites Amphibolites Quartz Porphyries
		unconformity	

Continued......

Table 1: Contd.

	Sakoli Group	Taffaceous shales B.H. Q. & Metabasic rocks.	Tuffaceous (?) shales B.H. Q. & Metabasic rocks.
Archean	Amgaon Group	Amgaon orogeny (c.2300 M A) Metamorphism and Granittsation (c. 2300 MA)	Not exposed

ultrabasic units are essentially concordant to the amphibolites and indicate synchronous emplacement of possibly cumulate origin. The younger intrusives (Gabbro and dolerite) are discordant. A major N-S trending fault (established by drilling) is present betwdeen the amphibolite and quartz porphyry. The major shear zones (N 18° E - S 18° W & N 40° W - S 40° E) host uranium mineralisation in both the areas. Along the transverese faults, conspicuous dykes of gabbro and dolerites are observed (Fig. 1).

2. Host rock petrography and nature of uranium mineralisation

The radioactive amphibolites show widely variable textural and structural characteristics. They range from fine to coarse grained, feeble to well foliated, meso to metanocratic and are often banded, Petrotgraphically, the amphibolites show nematoblastic schistose texture with hornblende, actinolite, biotite and mostly untwinned plagioclase feldspares as major constituents, but, with increase in biotite, chlorite and sericite, it tends towards lepidoblastic type. The non-schistose types on the other hand are massive medium grained and porphyritic to non- porphyritic types. The porphyritic varities have phenocrysts of plagioclase feldspars. The accessories are magnetite, ilmenite, pyrrhotite, pyrite, haematite, calcite and epidote, Sulphides occurring in amphibolite both in the mineralised as well as in their non-mineralised counterparts include pyrrhotite, marcasite, pyrite, polydymite, pentlandite, chalcopyrite, covellite and traces of sphalerite, galena and electrum.

Radioactivity is essentially confined to the dark bands rich in amphiboles (grunerite), biotite and opaque oxides along with some sulphides (Fig. 2, A,B & C). Ore microscopic studies have revealed that radioactivity is mainly due to the presence of uraninite as extremely small, irregular grains as well as very fine veins (few microns upto a few milimeters) that occupy grain boundaries and microfractures of amphiboles and biotite along the shear planes. uranium is also associated with haematite and /or goethite in an adsorbed state. Uranium notably occurs in a state of disequilibrium (15%) mostly in favour of parent elements and this state of disequilibrium is mainly because of transportation and migration of radium and radon.

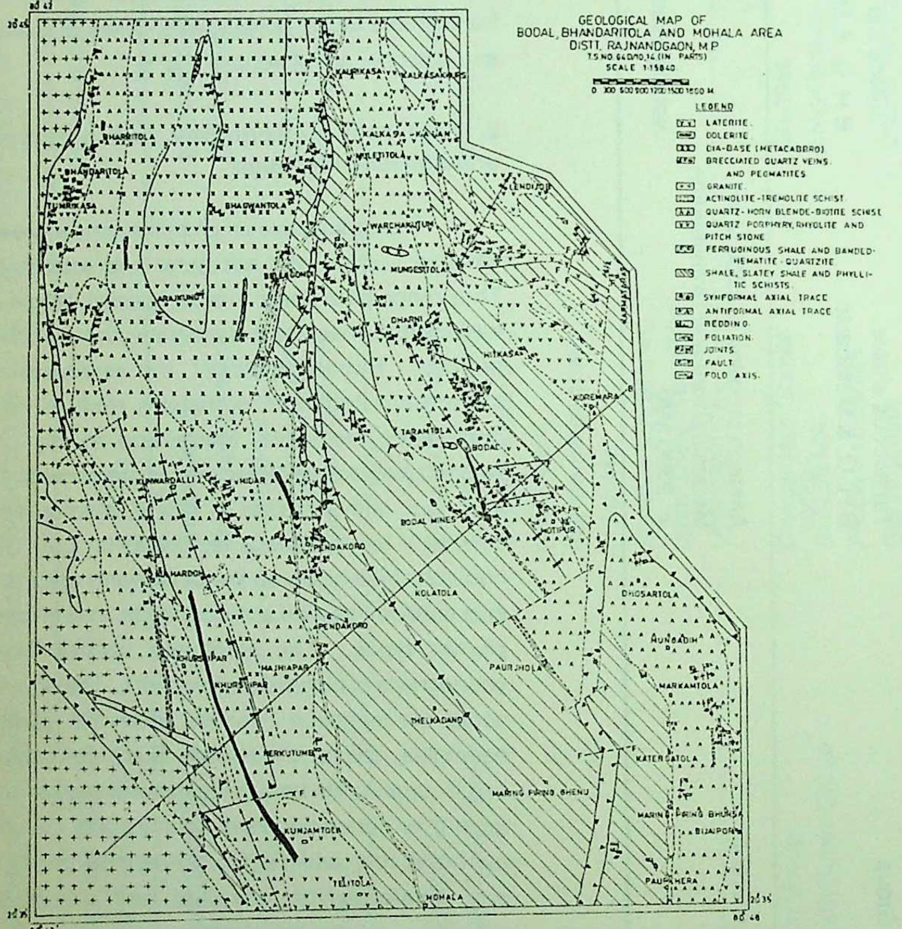
GEOLOGICAL MAP OF
BODAL, BHANDARITOLA AND MOHALA AREA
DISTT. RAJNANDGAON, M.P.
T.S NO. 64D/10,14 (IN PARTS)
SCALE 1:15840
0 300 600 900 1200 1500 1800 M
LEGEND
LATERITE
DOLERITE
DIA-BASE (METAGABBRO)
BRECCIATED QUARTZ VEINS AND PEGMATITES
GRANITE
ACTINOLITE-TREMOLITE SCHIST
QUARTZ-HORN BLENDE-BIOTITE SCHIST
QUARTZ PORPHYRY, RHYOLITE AND PITCH STONE
FERRUGINOUS SHALE AND BANDED-HEMATITE-QUARTZITE
SHALE, SLATEY SHALE AND PHYLLITIC SCHISTS
SYNFORMAL AXIAL TRACE
ANTIFORMAL AXIAL TRACE
BEDDING
FOLIATION
JOINTS
FAULT
FOLD AXIS
KALKASA
BHARRITOLA
BHANDARITOLA
TUMRIKASA
BHAGWANTOLA
WARCHAKUTUM
MUNGESTOLA
DHARNI
HITKASA
KOREMARA
TARAMTOLA
BODAL
BODAL MINES
MOTIPUR
PENDAKORO
KOLATOLA
DHOSARTOLA
MARKAMTOLA
PAURJHOLA
THELKADAND
KUNJAMTOLA
TELITOLA
MOHALA
PAUPAHERA
BIJAPOR
KHURSIPAR
MAJHIAPAR
NARDOH

Fig.1. Regional Geological Map of Boda-Bhandaritola area.

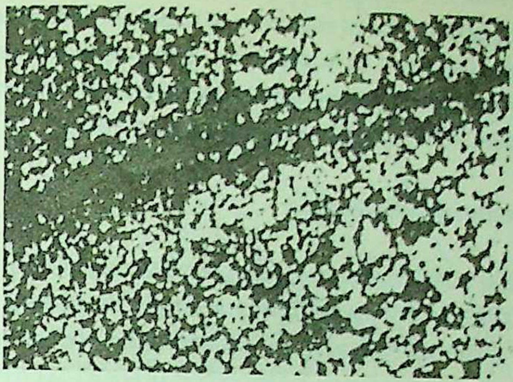

Fig. 2A. Uraninite and/or pitch blende occurs as minute black opaque globules or inclusions of less than one micron size in hornblende either disseminated or along microfactures and fracture planes under plane polarised light.

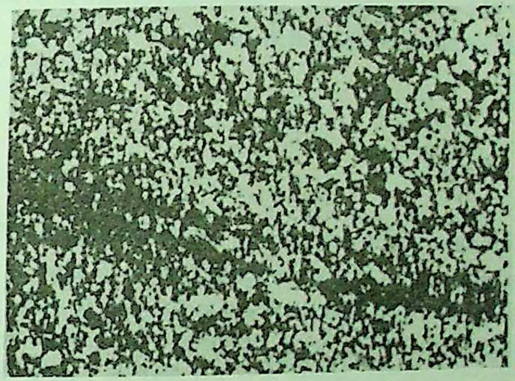

Fig. 2B. Same as above under crossed nicols

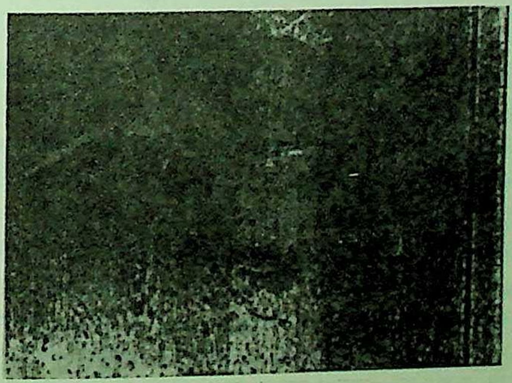

Fig. 2C. Alpha-Tracks along the fracture planes in amphi-bolite. Uraninite occurs as inclusions and/or globules in hornblende along the fracture planes.

Apart from the field evidence, laboratory studies also confirm hydrothermal emplacement oif uranium indicated by abundance of sulphides, silicification, chloritisation, sericitisation, calcitisation and the development of widespread biotite and fluorite. Although the mineralisation is structurally controlled, geochemical parameters have also played an equally important role in fixing uranium in the amphibolites (Sinha et al 1984).

3. Metamorphism and uranium mineralisation

The amphibolites have suffered with low grade of metamorphism of green schist and lower ambhibolite facies. Three metamorphic zones have been delineated, namely, chlorite, biotite and garnet, on the basis of mineralogical assemblages and isoreaction grades of amphibolites. Based on the studies of several metamorphic uranium deposits, it is seen thast the areas of lower metamorphic grade are comparatively richer because of strong adherence of uranium with H_2O molecules. In the present area, the grade of metamorphism is similarly low and hosts significant uranium mineralisation.

The inverse relation of the metamorphic grade with content of uranium and H_2O in these amphibolites has been established (Rai et al 1988).

4. Geochemistry

4.1 Analytical methods

Thirty and eleven representative samples of amphibolites from Bhandaritola and Bodal respectively were analysed for major oxides by rapid method of silicate analysis (Shapiro and Brannock, 1962 & Shapiro, 1975) except for Mg, Ca, Ti and Mn which together with the trace elements like Ni,Ba,Sr,Rb were determined by varion Atomic Absorption Spectrophotometer (AA-475). Trace elements (Cu, Pb and Zn) were determined by the spectrograph.

4.2 Chemistry of host rocks

The average compositions of unmineralised, feebly mineralised and mineralised ambhibolites of Bhandaritola and Bodal areas are given in Tables 2,2 a, b & c. Bulk chemical composition, normative computations and solidification index of these rocks indicate basalt to basaltic and site andesite nature of quartz-tholeiitic type (Krishnamurthy 1988). SiO_2 varies from 50.63 to 64.57% in amphibolites of Bhandaritola while 46.81 to 53.20% in Bodal indicating basalt to basaltic andesite composition in eastern part near Bodal while the basaltic andesite to andesitic composition occur near Bhandaritola located 10 kms. WNW of Bodal. Except a few samples of Bodal, majority of them are low mg basalts and are comparatively more enriched in the compatible elements (Gill,1981, Yoder & Tilley 1962, Poldervaart et al, 1964). Tholeiitic nature of the amphibolites is characterised by the iron enrichment as seen in the AFM diagram (Fig.3) and by an increase in $FeO + Fe_2O_3/MgO$ ratio with increasing SiO_2 content (Fig. 4), They show a fairly continuous compositional variation as indicated by

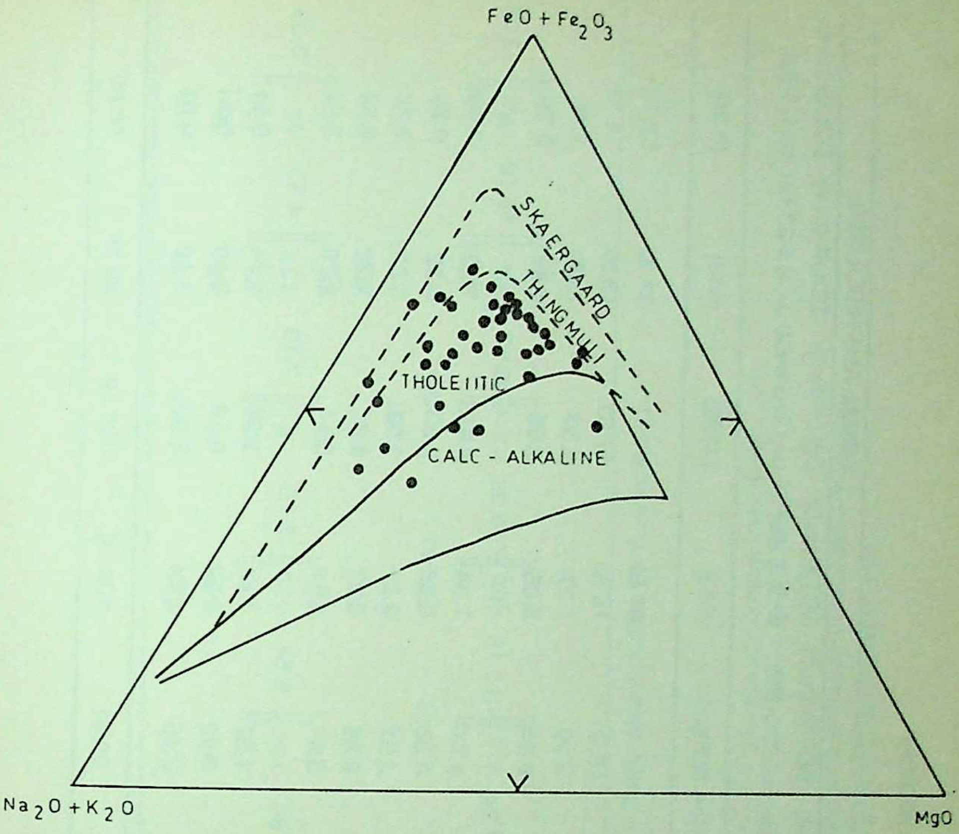

Fig.3. MgO-FeO+Fe$_2$O$_3$-Na$_2$O+K$_2$O diagram for the amphibolite of Bodal and Bhandaritola area .Field boundaries indicated are from Ringwood, 1975.

the variation diagram of SiO$_2$ Vs major oxides like FeO^T, MgO, K$_2$O, Na$_2$O, TiO$_2$, P$_2$O5, MnO, CaO, Al$_2$O$_3$ (Fig. 4) and trace elements like Cu, Pb, Zn, Ni, Ba, Sr, Rb (Fig. 5).

The decreasing trend of MgO, constant TiO$_2$ and initial high values of FeO^T (and probably of MnO also) upto a miximum of 57.0% SiO$_2$ is consistent with the tholeiitic trend. CaO concentrations also display the decreasing trend with a sharp depletion at more than 56% SiO$_2$. Al$_2$O$_3$ concentrations have a wide scatter. Contents of Na$_2$O and K$_2$O are distinctly much higher. Na$_2$O shows a slight increase, whereas K$_2$O bears no definite trend with the increase of SiO$_2$. P$_2$O$_5$ content displays some initial scatter and probably increases gradually with increase of SiO$_2$ upto 55% SiO$_2$ (Fig.4). Ni content is generally low (25 to 50 ppm) and shows decreasing tholeiitic trend while reverse is with Cu + Pb + Zn (185 to 316^o ppm). Pb content is extremely high (upto 3024 ppm) and it shows increasing trend with increase of SiO$_2$. Concentrations of Rb (13 to 88 ppm), Sr (270 to 750 ppm) and Ba (140 to 270 ppm) are high as expected with higher contents of K$_2$O and Na$_2$O (Rai et al,1988). Concentration of Sr and Ba varies widely and appears to increase with increasing SiO$_2$ while Rb content does not increase with increasing SiO$_2$ (Fig.5).

Table-2: Chemical composition of Bodal - Bhandaritiola Amphibolites

	BHANDARITOLA AMPHIBOLITES												BODAL AMPHIBOLITES					
Oxides	Standard deviation		BDT/A(n=30) u=20-8400		BDT/B(n=8) u=20-70		BDT/C(n=4) u=100-170		BDT/D (n=7) u=220-280		BDT/E(n=5) u=305-857		DBT/F(n=6) u=1310-8400		BDL/A (n=5) u=315-2290		BDL/B (n=6) u=1,5-8.2	
	(δ)		(990)		(44)		(126)		(254)		(583)		(4025)		(944)		(3.36)	
SiO_2	2.04		57.94		56.24		56.62		57.32		57.89		61.91		53.33		52.78	
Al_2O_3	0.84		16.28		15.99		15.16		15.81		16.63		17.63		13.22		14.12	
TiO_2	0.05		1.22		1 19		1.19		1.19		1.27		1.30		1.88		1.75	
Fe_2O_3	0.36	2.32	2.64	10.60	2.50	11.72	2.73	12.24	2.84	11.15	3.05	10.39	2.00	6.43	4.87	15.18	3.78	13.08
FeO	1.96		7.96		9.22		9.51		8.31		7.34		4.43		10.31		9.30	
MnO	0.01		0.05		0.05		0.06		0.06		0.05		0.03		0.22		0.24	
MgO	0.82		2.47		3.38		3.10		2.51		2.21		1.03		3.56		3.70	
CaO	0.42		3.12		3.19		3.30		3.35		3.47		2.31		5.70		8.52	
Na_2O	0.49		4.20	6.25	3.70	5.23	4.25	5.84	3.97	5.54	4.14	5.75	5.14	7.13	2.79	4.53	2.82	3.33
K_2O	0.23		2.05		1.53		1.59		1.57		1.61		1.99		1.74		0.51	
P_2O_5	0.07		0.27		0.34		0.39		0.32		0.24		0.19		0.40		0.34	
H_2O	0.20		2.31'		2.72		2.27		2.16		2.24		2.36		1.12		0.83	
Total			100.51		100.75		100.17		99.41		100.14		100.32		99.14		98.69	

Table-2 Contd. :

Pb	529	387	67	86	128	231	1444	476	112
Ni	1.6	42	41	40	44	44	42	40	38
Zn	15.2	130	129	149	136	111	109	93	106
Cu	41.9	85	38	91	154	120	57	137	54
Rb	7.5	35	46	32	27	26	38	36	19
Sr	66.8	488	377	568	536	532	490	488	315
Ba	146.2	1277	1490	1108	1140	1171	1351	1183	965
Cu+Zn	47.2	215.00	167.00	240.00	290.00	231.00	166.00	230	160
Ba+Rb+Sr	73.9	1800	1913	1708	1703	1729	1879	1707	1299
Cu+Zn+Pb	506.1	602.00	234.00	326.00	418.00	462.00	1610.00	706	272
Rb/Sr	0.09	0.07	0.12	0.06	0.29	0.23	0.12	0.07	0.06
Ba/Sr	0.73	2.62	3.95	1.95	2.13	2.20	2.76	2.42	3.06

1. BDT/B= Sample designation for 8 samples.
(n=8)
2. n=mumber of samples
3. $u = \frac{100 - 170}{(126)}$:Range of uranium from 100 to 170 with average of 126.
4. Trace elements and uranium values are in ppm.
5. Oxides in % and elements in ppm.

Tabel - 2(a): Chemical composition of Bhandaritola Amphibolites

Sample No.	U	SiO_2	Al_2O_3	TiO_2	Fe_2O_3	FeO	MnO	MgO	CaO	Na_2O	K_2O	P_2O_5	H_2O	Total
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
BDT-1	20	55.78	17.07	1.25	2.25	10.90	0.05	3.80	3.25	3.08	2.67	0.21	2.76	103.07
BDT-2	20	57.62	15.72	1.21	2.06	10.15	0.06	2.88	3.28	2.72	2.44	0.19	2.63	100.96
BDT-3	30	57.19	13.70	1.05	2.95	10.20	0.06	2.98	4.83	4.20	2.97	0.29	1.65	102.07
BDT-4	45	57.92	13.28	1.02	2.90	10.29	0.06	3.17	4.24	3.19	1.31	0.37	2.13	99.88
BDT-5	50	56.92	16.53	1.29	2.99	9.54	0.04	3.60	2.31	3.70	3.32	0.42	3.23	103.89
BDT-6	60	56.58	18.60	1.30	1.63	8.82	0.04	3.65	1.65	4.29	2.30	0.43	3.76	103.05
BDT-7	60	57.26	16.78	1.21	1.84	9.63	0.04	3.57	2.42	4.08	2.38	0.39	2.67	102.27
BDT-8	70	50.63	16.30	1.19	4.40	9.84	0.05	3.44	3.58	4.37	2.88	0.40	2.96	100.04
BDT-9	100	55.78	14.83	1.07	2.66	9.34	0.07	4.03	4.20	4.45	3.00	0.42	1.85	100.90
BDT-10	100	56.92	15.13	1.20	2.97	9.55	0.05	1.83	3.29	4.60	2.25	0.18	2.11	100.08
BDT-11	135	57.38	14.79	1.11	2.96	9.62	0.07	3.65	3.06	3.75	2.38	0.29	2.23	101.28
BDT-12	170	56.41	16.69	1.36	2.34	9.53	0.04	2.89	2.66	4.20	2.72	0.26	2.87	101.97
BDT-13	220	57.19	17.37	1.31	2.80	8.04	0.05	1.69	2.63	4.37	1.98	0.21	2.45	100.09
BDT-14	230	55.80	14.42	1.07	3.03	8.42	0.07	4.15	4.56	4.25	1.89	0.87	1.76	100.29
BDT-15	245	56.85	16.53	1.23	2.87	8.54	0.06	2.43	3.64	4.34	1.08	0.23	2.30	100.10
BDT-16	260	57.73	14.03	1.07	3.86	8.20	0.07	2.68	3.98	3.55	1.86	0.27	1.56	98.86

Table-2(a) Contd.

BDT-17	270	57.42	14.60	1.12	2.54	8.34	0.07	3.44	3.79	3.75	0.79	0.35	1.72	97.93
BDT-18	270	59.12	14.63	1.34	2.93	8.39	0.04	1.33	2.50	4.47	2.43	0.15	2.52	102.85
BDT-19	280	57.13	17.14	1.21	2.00	8.26	0.05	1.83	2.36	3.09	2.98	0.19	2.79	99.03
BDT-20	305	58.21	18.53	1.32	1.86	8.01	0.04	2.90	2.49	5.50	1.76	0.28	2.07	102.97
BDT-21	440	58.54	15.05	1.14	3.42	7.20	0.06	1.96	3.47	3.62	0.79	0.20	1.43	96.88
BDT-22	500	57.10	16.75	1.30	3.30	7.10	0.05	1.99	2.44	3.64	2.73	0.21	3.26	99.87
BDT-23	815	57.87	19.39	1.52	3.43	7.34	0.04	0.71	4.85	3.44	1.76	0.17	2.56	100.08
BDT-24	857	57.67	13.42	1.07	3.23	7.05	0.07	3.48	4.10	4.50	1.02	0.35	1.87	97.83
BDT-25	1310	58.69	19.05	1.44	2.46	5.09	0.04	1.45	1.98	5.26	2.37	0.18	2.78	100.79
BDT-26	1320	59.51	18.11	1.29	2.00	5.88	0.03	1.73	2.74	5.20	1.82	0.23	2.43	100.97
BDT-27	2425	63.23	16.42	1.22	2.24	4.67	0.04	0.87	2.21	4.87	1.79	0.13	2.18	99.87
BDT-28	2600	61.05	16.10	1.30	3.59	4.75	0.03	0.78	2.23	4.50	2.94	0.16	2.12	99.55
BDT-29	8095	64.43	18.00	1.31	0.70	2.81	0.02	0.71	2.26	5.65	1.95	0.26	2.76	100.86
BDT-30	8400	64.57	18.11	1.22	1.02	3.37	0.03	0.61	2.42	5.36	1.07	0.18	1.89	99.85
X	990	57.94	16.28	1.22	2.64	7.96	0.05	2.47	3.12	4.20	2.05	0.27	2.31	100.51

Table - 2(b): Chemical compositoin of Bhandaritola Ampihibolite

Sample No.	U	Pb	Zn	Cu	Ni	Rb	Sr	Ba
BBT-1	20	45	130	55	45	51	420	2220
BDT-2	20	35	130	30	45	40	375	1870
BDT-3	30	45	135	15	45	21	475	650
BDT-4	45	50	110	15	45	27	520	1120
BDT-5	50	155	185	00	35	37	275	1475
BDT-6	60	55	110	25	25	80	285	1000
BDT-7	60	55	105	45	40	48	270	975
BDT-8	70	95	125	40	50	51	395	1010
BDT-9	100	76	145	95	45	23	595	1150
BDT-10	100	67	140	75	40	25	595	1050
BDT-11	135	90	150	130	40	23	640	860
BDT-12	170	110	160	65	35	56	440	1370
BDT-13	220	110	105	170	45	35	440	1330
BDT-14	230	155	185	220	50	27	540	755
BDT-15	245	120	145	440	30	21	750	785
BDT-16	260	110	125	85	40	14	560	780
BDT-17	270	120	125	50	50	13	605	950
BDT-18	270	145	140	85	40	34	480	1580
BDT-19	280	135	130	30	50	45	375	1800
BDT-20	305	130	70	80	45	38	475	1390
BDT-21	440	165	110	160	35	16	715	705
BDT-22	500	190	145	90	45	17	365	1540
BDT-23	815	300	65	50	45	33	510	1370
BDT-24	857	369	165	220	50	28	595	770
BDT-25	1310	400	100	50	35	49	555	1430
BDT-26	1320	495	150	70	35	37	570	1500
BDT-27	2425	900	135	70	50	42	455	1280
BDT-28	2600	940	125	75	45	50	350	1540
BDT-29	8095	2826	130	45	40	13	505	1320
BDT-30	8400	3024	115	30	45	26	505	955

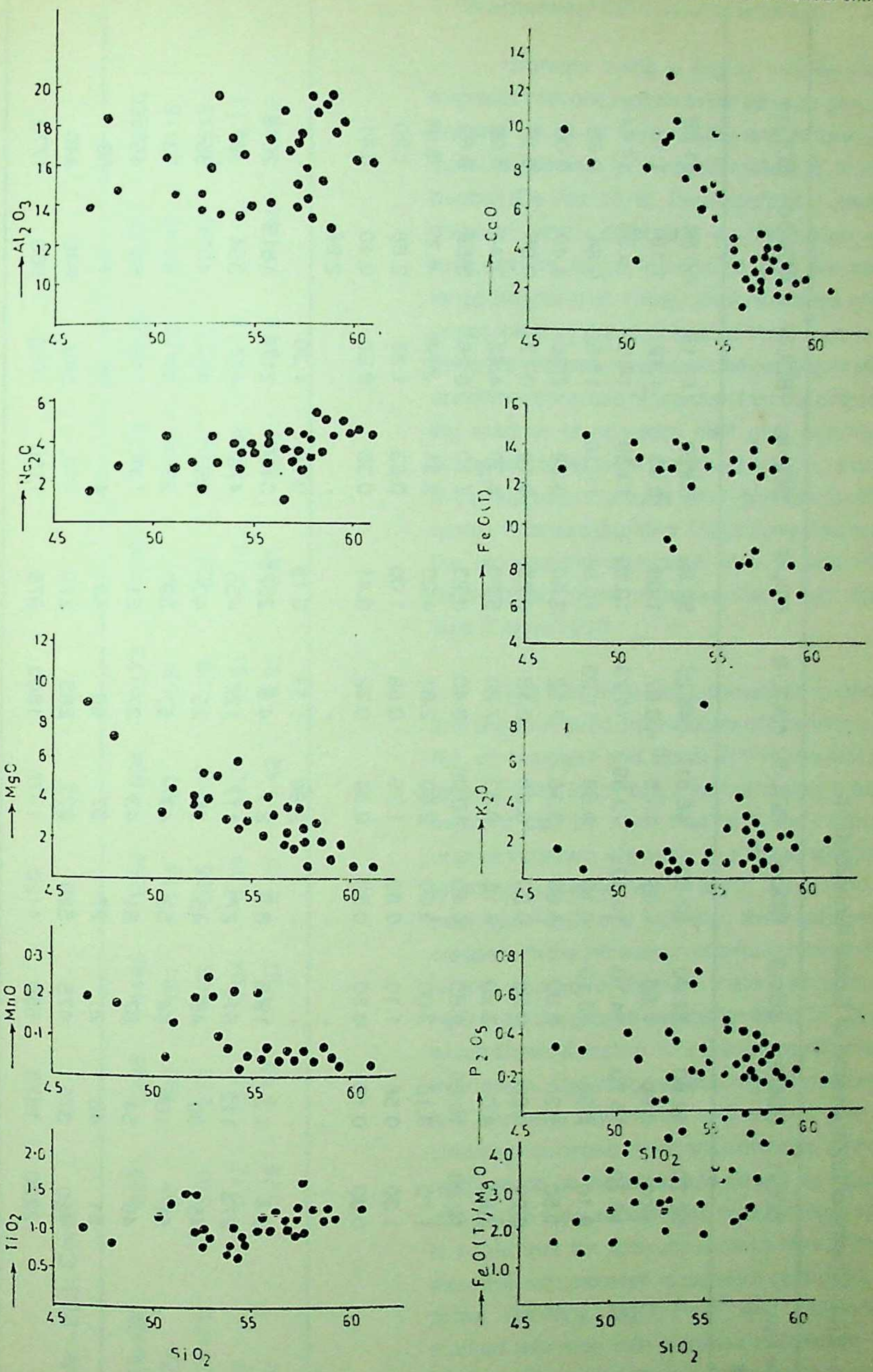

Fig.4. SiO_2 Vs major oxides of meta-basalts and meta-andesites.

Table - 2(c): Chemical composition of Bodal Amphibolites

Sample No. Oxides	BDL-1	BDL-2	BDL-3	BDL-4	BDL-5	BDL-6	BDL-7	BDL-8	BDL-9	BDL-10	BDL-11
SiO_2	52.50	52.80	52.40	52.20	46.81	48.25	52.88	53.20	51.15	50.63	51.28
TiO_2	1.00	0.80	1.50	1.05	1.05	0.87	1.08	0.95	1.42	1.19	1.13
Al_2O_3	13.70	15.70	14.10	14.30	13.89	14.73	13.59	13.59	14.50	16.30	14.44
FeO	8.10	7.90	10.10	8.10	9.58	11.23	10.80	11.16	11.52	9.84	9.83
Fe_2O_3	1.50	1.20	3.00	4.80	3.36	3.52	3.60	3.20	2.26	4.40	3.08
MnO	0.20	0.20	0.20	0.20	0.20	0.18	0.25	0.10	0.13	0.05	0.17
MgO	3.80	4.00	3.20	4.10	8.83	7.20	5.30	5.30	4.42	3.44	4.96
CaO	12.90	10.60	9.80	9.40	10.08	8.40	8.23	7.70	8.26	3.58	8.89
Na_2O	1.70	3.10	3.00	3.00	2.60	2.81	4.25	3.31	2.68	4.37	3.08
K_2O	1.30	0.58	1.10	0.80	1.75	0.66	1.00	0.63	1.35	2.88	1.20
P_2O_5	0.80	0.18	0.50	0.50	0.34	0.32	0.41	0.36	0.28	0.40	0.41
H_2O	-	-	-	-	-	-	-	-	-	2.96	-
LOI	-	-	-	-	2.66	1.54	0.75	0.78	1.30	-	-
U	3.4	1.2	19.8	8.5	2.7	4.5	2525	3400	2176	1815	2615
Pb	273	112	315	271	117	125	435	476	427	354	397
Ni	45	38	45	45	35	25	40	45	40	40	35
Zn	51	106	64	53	47	67	39	93	37	59	73
Cu	46	54	87	61	43	37	51	137	118	84	45
Rb	51	43	21	27	57	88	49	43	75	73	56
Sr	420	375	475	520	272	285	270	485	595	640	440
Ba	2220	1870	650	1120	1475	1800	975	1150	1050	860	1770

Table - 2(c): Contd.

Ba+Rb+Sr	2691	2288	1146	1667	1804	2173	1294	1678	1720	1573	2266
Cu+Zn	97	160	151	114	90	104	90	230	155	143	118
Cu+Zn+Pb	370	272	466	385	207	229	525	706	582	497	515
Rb/Sr	0.12	0.11	0.04	0.05	0.21	0.31	0.18	0.09	0.13	0.11	0.13
Ba/Sr	5.29	4.99	1.37	2.15	5.42	6.32	3.61	2.37	1.76	1.34	4.02

5. Geochemical behaviour of uranium

Uranium being a highly mobile element migrates from one system to the other on geochemical and structural favourability and is fixed in the suitable locales forming ultimately a workable deposit (De Voto 1978). Two important properties of uranium, which determine its distribution in nature.,are its large ionic size and its polyvalence(Adams et al, 1959).Uraniu therefore remains unaccepted by the structure of many rock forming minerals and gets enriched in the residual mellt. This enrichment process of uranium fromthe differentiating magma or anatectic melt gets accumulated along with other large ion lithophile (LTL) elements in the hydrothermal fluids which give rise to different types of uranium deposits. This is fairly matched with the process of mobilisation of U,Th and Rb and redistributed in the metasomatising hydrothermal fluid (Cortini 1984).

The chemical composition of mineralised and unmineralised amphibolites of Bhandaritola (30 No. of samples) and Bodal (11 No.samples) are given Tables 2, 2a, b & c for comparative assessment. It can be seen that with the increase in uranium content, elements like silica, alkalies and alumina increase while MgO, MnO and FeO decrease.Fe_2O_3 and TiO_2 also show consistent increase with the increase in uranium content. Notably, with increase of uranium, there is an increase in H_2O in all the groups except for the BDT - 2 group which shows a decline in H_2O content with increase in uranium. This indicates that higher uranium content coincides with higher H_2O content only in the chlorite or chlorite-biotite transition zone. While TiO_2 is known to be immobile and shows no significant correlation with uranium, yet, in the present study, it is noted that the titanium content also shows an increase with increase in uranium content in these rocks. Similarly, MgO, FeO^T and FeO show a marked depletion with increase in uranium. CaO, MnO and P_2O_5 also show a decrease but relatively less to the former sets. Therefore, these compatible

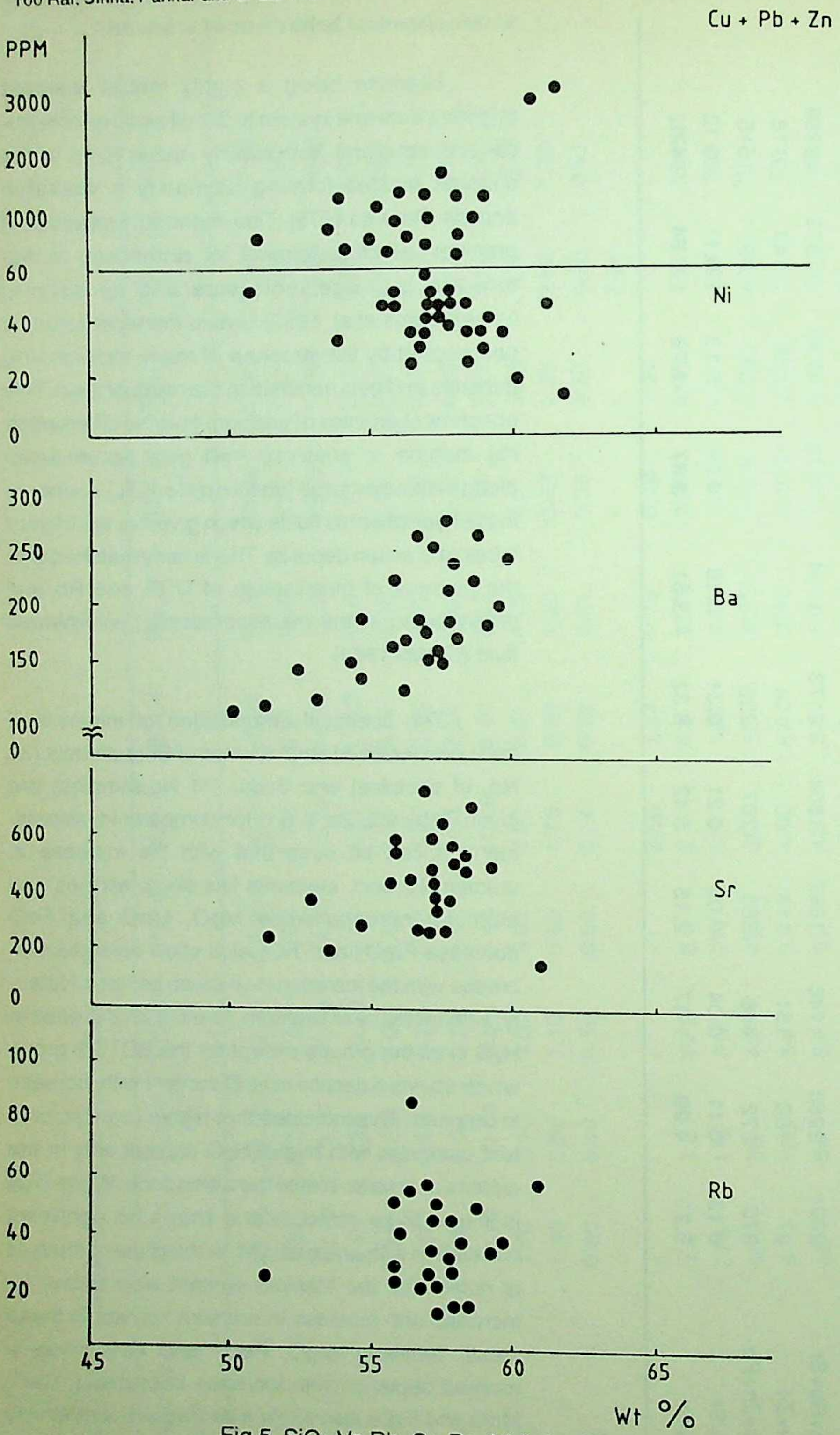

Fig.5. SiO2 Vs Rb, Sr, Ba, Ni & Cu+Pb+Zn.

elements except TiO_2 show negative correlation with uranium(Table 3, Fig. 6 A & B).

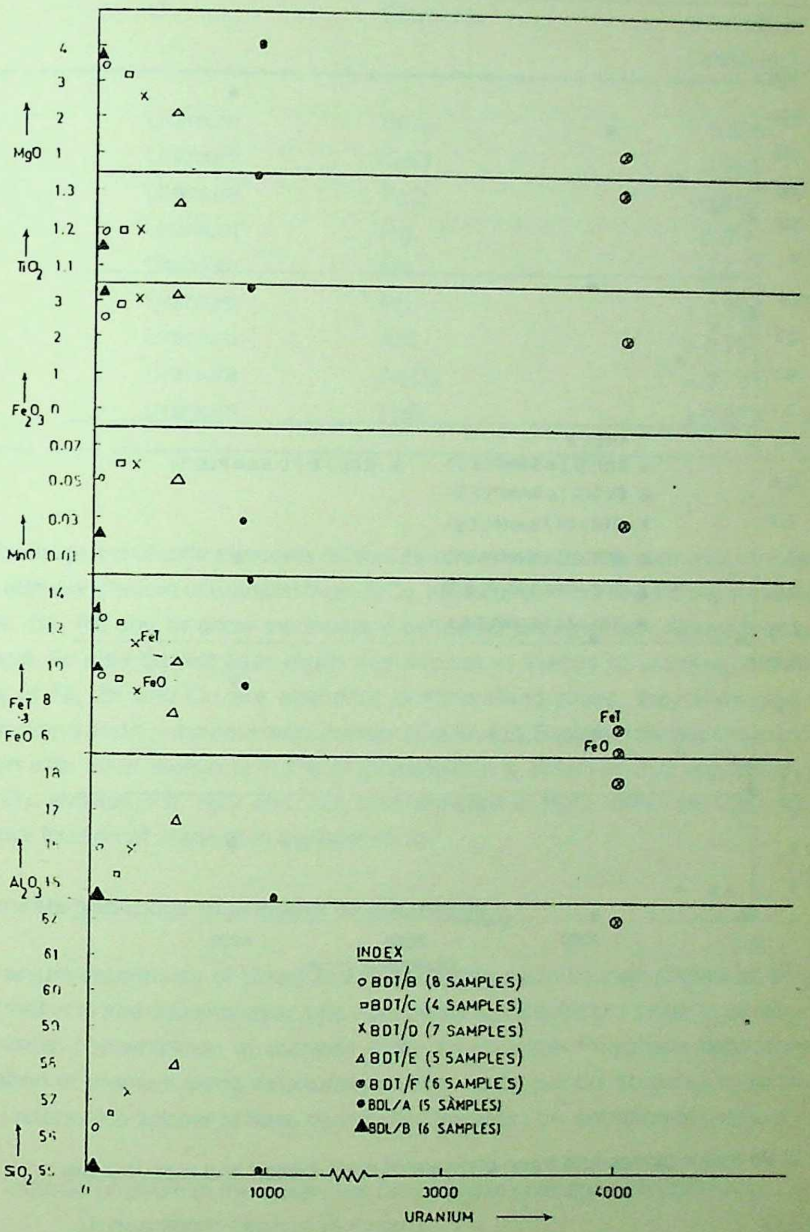

Fig 6A. U Vs major oxides of meta-basalts and meta- andesites.

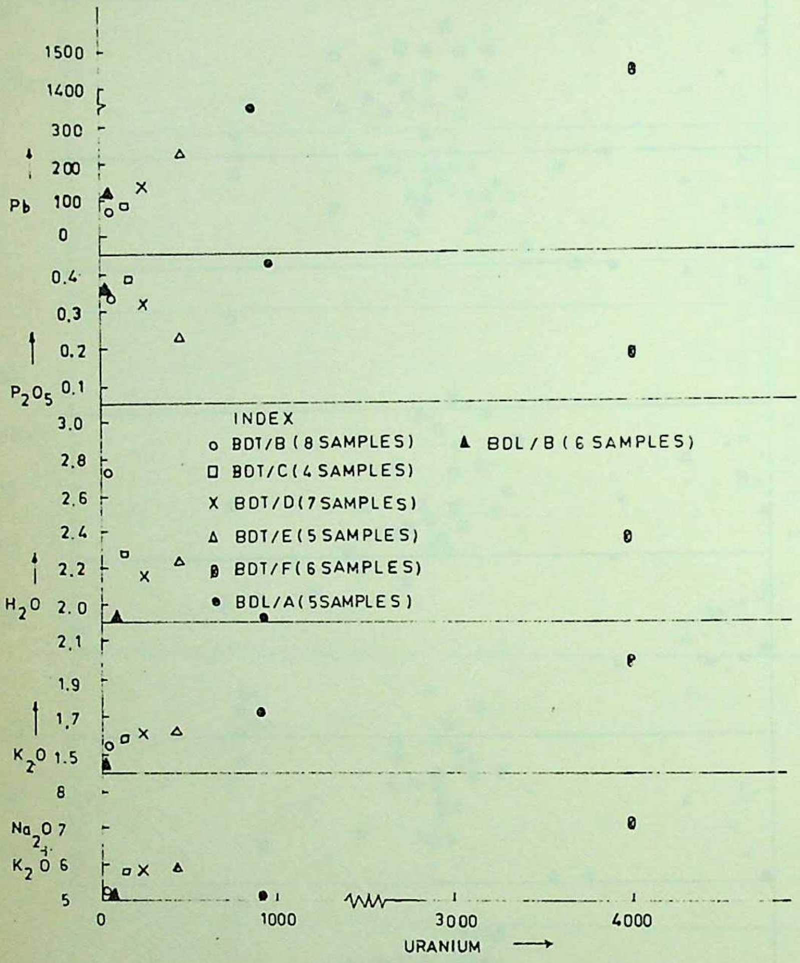

Fig.6B. U Vs major oxides and trace element of meta-basalt and metarhyolite.

Table-3: Correlation coefficient of uranium with different elements

S.No.	U	Elements	Correlation Coefficient (r)
1	Uranium	SiO_2	+ 0.896
2	Uranium	CaO	- 0.968
3	Uranium	FeO	- 0.695
4	Uranium	Hg	- 0.6
5	Uranium	Mn	- 0.6
6	Uranium	Pb	+ 0.99
7	Uranium	AlK	+ 0.79
8	Uranium	Al_2O_3	+ 0.731
9	Uranium	H_2O	+ 0.673

The large ion lithofile elements as well as some incompatible elements of these rocks increase with enrichment of uranium thus, SiO_2, alkalies, alumina and Pb are enriched in the ore zones. Ba, Rb and Sr show inconsistent behaviour with uranium. Normalised ratios of Ba, Rb and Sr also do not bear much significance in fixation of uranium. Although the sulphides of Ni, Zn and Cu are abundant in mineralised zones, they show and overall inconsistent and eratic behaviour with uranium (Fig.6A &B). Based on the geochemical affinity of uranium with other elements in the amphibolites, it is observed that with enrichment of SiO_2, Al_2O_3, alkalies, Pb, H_2O and TiO_2 and depletion in MgO, total iron, CaO, MnO and P_2O_5 favour fixation of uranium in the host rocks.

6. Ore Genesis and mode of uranium emplacement

Uranium ocurrences of Bodal and Bhandaritola could be categorised as epigenetic hydrothermal vein and disseminated type. Apparently, several factors seem to be responsible for preferential concentration of uranium in the amphibolites Polyphase deformation and concentration of uranium along structurally weaker zones are conspicuous In addition, the following factors also appear to have contributed towards concentration of uranium.

1. Intrinsic uranium in the basement Sakoli shale (average of 5 ppm)and Dongaragarh Granite (4-8 ppm).
2. Hybridization of amphibolites due to assimilation of rhyolite/quartz porphyry, etc. which contained conspicuously higher uranium content.
3. Tectonism accompanying granitisation triggering low grade metamorphism of amphibolites.

4. Impress of uranium being pneumatolytic phase of Dongargarh granite.
5. Intense shearing.

7.Conclusion

Based on studies as above ,it is concluded that apart from the structural control, various geochemical parameters have played an important role directlly or indirectly in fixing uranium in the amphibolites.Thus,it has been suggested that the rocks enriched in Si, Al, total alkalies, Pb and H_2O and depleted in Mg, Ca, FeO, Mn and P_2O_5 may be uranium-bearing, even though the surface manifestations of radioactivity are lacking, and these elements could be taken as reliable pathfinders for search of uranium in amphibolites in similar geological setting.

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Pink Granite Intrusion and Migmatisation of Two-Pyroxene Granulites in and around Manalur, South and North Arcot Districts, Tamilnadu, India

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Abstract. The intrusion of pink granite (2380-2000 m.y.) and its consanguineous pegmatites in and around Manalur,possibly resulted in the migmatization of the two-pyroxene granulites.It is presumed that the reactive fluids had moved along the major shear planes of Eastern Ghat belt and Caused the formation of intermediate and acidic rocks.Petrochemical study of the different rock types also confirm this view.

1. Introduction

The migmatitic rocks in and around Manalur (Map) are predominantly non-homogeneous.While the enclosing country rock (paleosome, after Mehnert, 1968) is two-pyroxene basic granulites-referred to as "two-pyroxene granulites"- in this paper ,the other are identified to be pink granites and its consanguineous pegmatites(neosome).The rocks of mixed origin aredesignated as migmatites. which are of intermediate and acidic in composition. These units often exhibit imperceptible gradations in the field.

2. Field relations

Migmatites show granular, gneissic, medium-grained, coarse- grained and phenoblastic texture. A particular type of migmatite does not exist in a region to the sole exclusion of other types.

The granular Variety is the most predominant one. The gneissic character is developed due to parallel alignment of dark minerals in thin foliae.The foliae are not entirely even,but thicken and thin out irregularly.The granular variety is dark greyish in colour and consists mainly of feldspars.The phenoblastic variety consists of phenoblasts of feldspars surrounded by smaller grains of feldspars. The potash feldspar (phenoblast) imparts a schlieren structure to the rock.

The two-pyroxene granulites are observed as patches (Harley,1989) or enclaves (Saravanan and Ramanathan,1978) or restites (Kuity,1990) in the migmatites. When these clots assume leans shape, the longer axes of the lens are parallel to the strike of foliation of the adjoining formations.Various stages of migmatisation of the original ortho two-pyroxene granulites (paleosome) by the later granites and its consanguineous pegmatites (neosome, after Mehnert,1968) can be traced in the area.

The first stage appears to be the injection of the gramitic/pegmatitic fluids along the weak planes of the paleosome (Fig.1) forming crude foliae of alternate light and dark bands. With the intensification of the process these acid foliae not only widen,but also in places,cut across the paleosome and enclose them as clots and pods (Fig.2) imparting an agmatic structure. During such process, the neosome appears to have produced retrogressive effects, along the contacts of paleosome (Ramanathan,1981 and Harley,1989).

Much of the two-pyroxene clots show kelyphitic sums, that is, they are amphibolized along their borders, leaving behind a kernel of unaltered portion.The altered (amphibolised) borders are in turn rimmed by a thin zone of biotite. A rich concentration of sphene occurs in both the hornblende and biotite layers.The occurrence of sphene is attributed to the richness of paleosome in calcium.(Milton,1960 and Ramanathan,1981).

3. Petrography

In migmatites the leucosomes are composed of crystalloblasts of microcline and oligoclase and interstitial quartz. The potash feldspar is perthitic to microperthtic. Quartz is ubiquitous and often bear strain shadows. They occur either as large irregular independent grains or as small xenomorphic grains in quartz-feldspar masaic. Apatite and sphene occur as accessories.

Table - 1(a): Petrochemistry in the area

	Two-pyroxene Granulite A	Amphibolised Granulite B	Migmatite (Average of 5) C- D	Pink Granite E
SiO_2	49.61	51.61	60.58	70.55
TiO_2	0.42	0.70	-	tr
Al_2O_3	20.18	18.31	15.83	12.43
Fe_2O_3	1.49	2.36	3.36	3.20
FeO	8.36	8.23	4.46	1.59
MnO	0.15	tr	-	tr
MgO	5.20	4.33	2.91	1.23
CaO	10.94	7.10	4.69	1.96
Na_2O	3.08	2.12	2.49	2.68
K_2O	0.60	4.70	5.70	6.33
P_2O_5	0.20	0.52	-	-
H_2O	0.28	0.50	n.d	n.d
	100.51	100.48	100.02	99.97

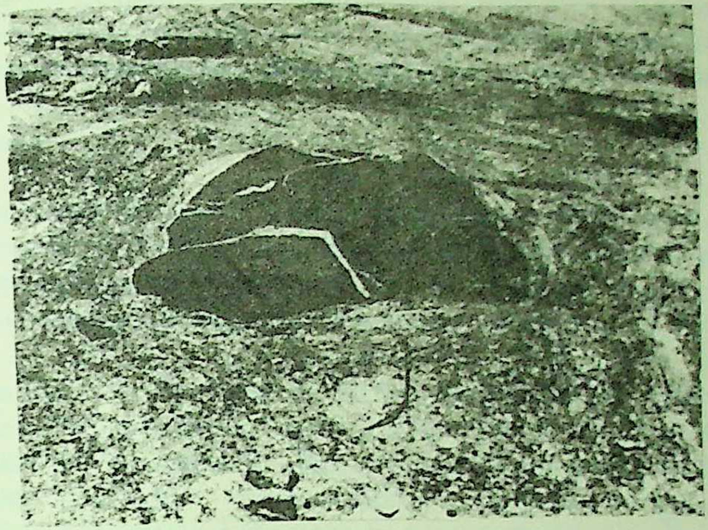

Fig. 1: Thin acid venis penetrating the weak planes of the paleosome.

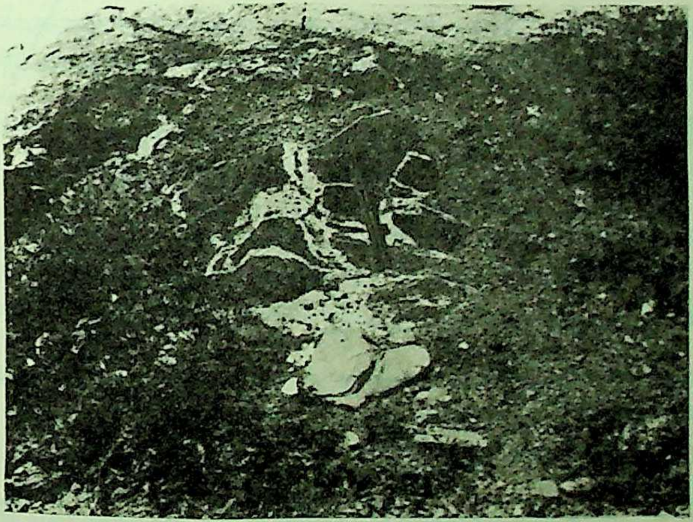

Fig. 2: Venis cutting across the paleosome, imparting an agmatic structure.

The melanosome exhibits a fine-grained fabric of mainly relict characters.The minerals in melanosome in order of increasing abundance are: hypersthene, diopside and hornblende. The latter when intergrown with either hypersthene or with diopside shows replacement relationship .

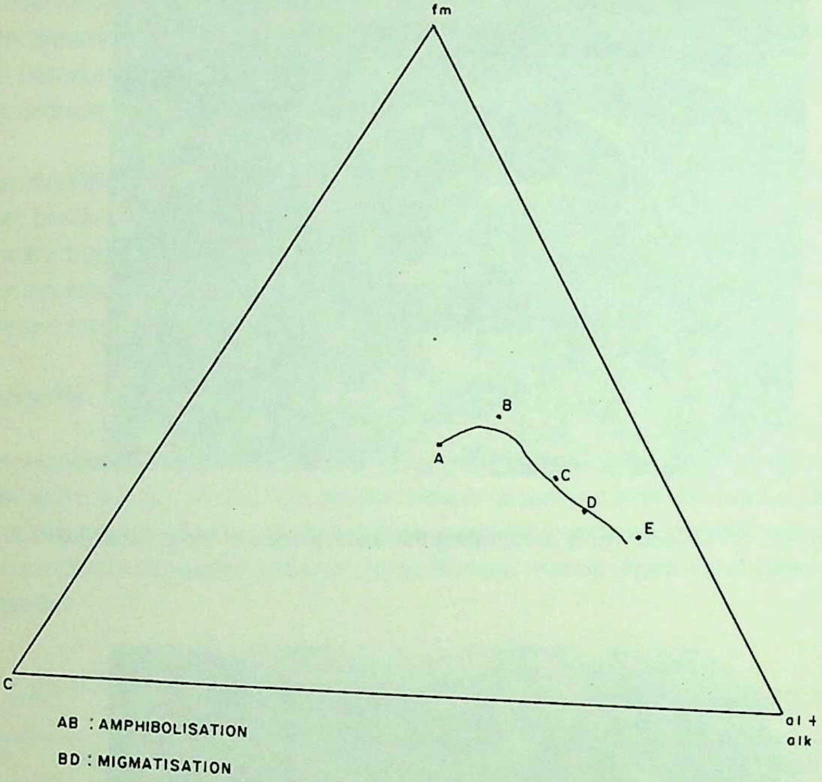
fm
B
A
C
D
E
C
al +
alk
AB : AMPHIBOLISATION
BD : MIGMATISATION

Fig. 3: The trilinear coordinate diagram.

Table - 1(b): Modal abundance of the minerals.

	Mode			
	A	B	C - D	E
Quartz	-	2.52	7.06	30.13
Feldspars	69.70	50.59	60.94	63.74
Hypersthene	11.10	10.32	10.72	-
Diopside	16.30	13.79	11.28	-
Hornblende	-	18.23	8.11	-
Biotite & Accessories	2.90	4.55	1.89	6.13

4. Petrochemistry

The wet analyses of migmatites (Table-1a) clearly show that these are chemically intermediate in comopósition between two- pyroxene granulite and granites. The analyses also indicate that there is a wide variation in SiO_2 content, and also that are more mobile.

Conclusions

The two-pyroxene granulites occur, as patches of various shapes and sizes in the migmatitic rocks. The mixing up of the two units in various proportions gave rise to gradational rocks containing felsic and mafic minerals. Among the various types of migmatites, one unit may be seen to pass gradually into another over some distance.

The field evidences suggest the fact that granite-pegmatite is the "granitizing agent" which caused all these transformations. Perhaps, the process of granitiation tends to eliminate all differences between the bulk composition of primary rocks and the final product, which is often granite itself. This leads support to the assumption that granites and its consanguineous pegmatite are responsible for the migmatisation of two-pyroxene granulites with the production of intermediate and acidic rocks.

The probable stages of formation of migmatitic rocks are:

1. Stage prior to metatexis: A stage where the paleosome and neosome constitute separate units or having clear contacts (Kuity, 1990)

2. Metatexis stage: Represents the incipient and partial melting stage when paleosome and neosome can still be distinguished petrographically.

3. Inhomogeneous diatexis stage: This represents the still advanced stage in migmatisation, which is marked by a complete or nearly complete mining of paleosome and neosome.

A trilinear co-ordinate diagram (after Harry,1952) based on Niggli values with fm, C and al+alk as end members illustrating the migmatisation (Fig.3) shows that the trend AB indicates amphibolisation followed by migmatisation, BD trend line.

In the area (map) the seat of intrusion of the pink granite is towards the southwestern portion. The mobilizates, therefore, had possibly moved from the southwest to northeast, along the structural weakness (Kuity,1990) viz., Eastern Ghat trend (2500 M.Y. Sarkar, 1968). As a result, extensive migmatisation of earlier two-pyroxene granulites had occurred.

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MHD unsteady free convection flow with mass transfer through a porous medium with variable suction

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Abstract. In this paper the effects of mass transfer on unsteady free convection flow of an incompressible electrically conducting viscous liquid through a porous medium having variable suction bounded by a vertical infinite plate has been discussed. The expressions for the velocity, temperature and mass concentration field have been obtained in two cases. The flow phonomena have been characterized by different non- dimensional parameters P,Gr, Gm, M,K, Sc and W. The effects of Sc on velocity profile have been shown graphically for different times. The effects of other parameters have also been discussed quantitatively on velocity and temperature field.

1.Introduction

Flows caused due to differences in concentration and in conjuction with temperature differences have great importance due to thier applicetion in the study of geophysics, aeronautics and engineering. Many authors have paid their attentions towards the study of such problems. First of all Kafousias and Raptis(1981) discussed the mass transfer effects on the unsteady free convection flow of an incompressible viscous fluid past an infinite vertical accelerated porous plate. After that Raptis et al. (1981 a & b, 1982) have studied the steady free convection and mass transfer through a porous medium. Mohapatra and Senapati (1984) have also discussed the steady MHD free convection flow through a porous medium with mass transfer. After that Raptis and Vlahos (1982) have considered the problem of unsteady free convection flow though a porous medium in the presence of a constant magnetic field.

Recently Mohapatra and Senapati (1987) have discussed MHD unsteady free convection flow with mass transter through a porous medium. So the aim of the study is to discuss Mohapatra and Senapati"s problem for variable suction. The problem has been studied in two cases. The expressions for the velocity, temperature and mass concentration field have been obtained in these cases. The effects of Sc have been shown graphically on velocity profile at different times. The effects of other parameters have also been discussed quantitatively.

2 Formulation of the problem

Here x-axis is taken along the vertical plate in the upward direction and y-axis is considered normal to it. Since the plate is infinite in length all the variables here are functions of y and t only. An uniform magnetic field of strength B_o is applied perpendicular to the plate and there is a variable suction at the plate in the negative direction of y-axis. With the usual Boussinesq approximations the governing equations for the considered problem are .

$$\frac{\partial v^x}{\partial y^x} = o \tag{2.1}$$

$$\frac{\partial u^x}{\partial t^x} + v^x \frac{\partial u^x}{\partial y^x} = \nu \frac{\partial^2 u^x}{\partial y^{x^2}} - \frac{\nu}{k^x} u^x - \frac{\sigma B_o^2}{\rho} u^x + g\,\beta\,(T^x - T^x\infty) + g\,\beta^x (C^x - C^x\infty) \tag{22}$$

$$P\,C_P\left(\frac{\partial T^x}{\partial t^x} + u^x \frac{\partial T^x}{\partial y^x}\right) = k \frac{\partial^2 T^x}{\partial y^{x^2}} \tag{2.3}$$

$$\frac{\partial C^x}{\partial t^x} + u^x \frac{\partial C^x}{\partial y^x} = D \frac{\partial^2 C^x}{\partial y^{x^2}} \tag{2.4}$$

Here all the symbols have their usual meanings. Form the equations of continuity (2.1) we take

$$v^x = -v_0[1 + \varepsilon f(t^x)] \tag{2.5}$$

when $\varepsilon < 1$

Particular Cases

(1) Considering $f(t') = e^{iw't}$ (2.6)

Introducing the following non-dimensional quantities

$$Y = \frac{Y^x v_o}{\nu},\ t = \frac{t^x v_o^2}{\nu},\ u = \frac{u^x}{v_o},\ T = \frac{T^x - T^x_\infty}{T^x_w - T^x_\infty}$$

$$C = \frac{C^x - C^x}{C^x_w - C^x_\infty},\ K = \frac{v_o^2}{\nu^2} k^x,\ M = \frac{\sigma B_o^2}{p u_o^2},\ P = \frac{p\,\nu\,C_p}{k} \tag{2.7}$$

$$Sc = \frac{\nu}{D},\ Gr = \frac{g\,\beta\,\nu\,(T^x_w - T^x_\infty)}{v_o^3},\ Gm = \frac{g\,\beta^x\,\nu\,(C^x_w - C^x_\infty)}{u_o^3},\ W = \frac{\nu\,w^x}{v_o^2}$$

The above equations take the following forms

$$\frac{\partial^2 u}{\partial Y^2} + [1 + \varepsilon\,e^{iwt}]\frac{\partial u}{\partial Y} = \frac{\partial u}{\partial t} + \left(M + \frac{1}{k}\right) u - GrT - GmC \tag{2.8}$$

$$\frac{\partial^2 T}{\partial Y^2} + P\,(1 + \varepsilon\,e^{iwt})\frac{\partial T}{\partial Y} = P\frac{\partial T}{\partial t} \tag{2.9}$$

$$\frac{\partial^2 C}{\partial Y^2} + Sc\,(1 + \varepsilon\,e^{iwt})\frac{\partial C}{\partial Y} = Sc\frac{\partial C}{\partial t} \tag{2.10}$$

The equations (2.8) to (2.10) are to be solved under the following boundary conditions.

$$T = 1 + \varepsilon\,e^{iwt},\ C = 1 + \varepsilon\,e^{iwt},\ u = o \ \text{at}\ Y = o$$

$$T \to 0,\ C \to 0,\ u \to o \ \text{as}\ Y \to \infty \tag{2-11}$$

3. solution of equations

For solving above equations subject to the boundary conditions (2.11) we assume that

$$\left.\begin{aligned} u(Y,t) &= u_o(Y) + \varepsilon u_1(Y) e^{iwt} \\ T(Y,t) &= T_o(Y) + \varepsilon\,T_1(Y)\,e^{iwt} \end{aligned}\right| \tag{3.1}$$

$$C(Y, t) = C_o(Y) + \varepsilon u_1(Y)e^{iwt}$$

with the convention that only the real parts of the complex quantities have physical significance in our problem.

Substituting (3.1) in (2.8) - (2.10) and comparing the harmonic terms we get

$$\ddot{u_o} + \dot{u_o} - (M + \frac{1}{k})\, u_o = -Gr\, T_o - Gm\, C_o \tag{3.2}$$

$$\ddot{u_1} + \dot{u_1} - (M + \frac{1}{k} + iw)\, u_1 = -u'_o - Gr\, T_1 - Gm\, C_1 \tag{3.3}$$

$$\ddot{T_o} + P\dot{T_o} = o \tag{3.4}$$

$$\ddot{T_1} + P\dot{T_1} - piw\, T_1 = p\, \dot{T}\, o \tag{3.5}$$

$$\ddot{C_o} + Sc\dot{C_0} = 0 \tag{3.6}$$

$$\ddot{C_1} + S\dot{c_1} - iw\, ScC_1 = -C'_o Sc \tag{3.7}$$

where primess denote differentiation with respect to Y. The boundary conditions for u_o, u_1, T_o, T_1, C_o and C_1 are as follows :

$$\left.\begin{array}{l} u_o = o,\ u_1 = o,\ T_o = 1,\ T_1 = 1,\ C_o = 1,\ C_1 = 1 \text{ at } Y = 0 \\ u_o \to o,\ u_1 \to 0,\ T_o \to 0,\ T_1 \to 0,\ C_o \to 0,\ C_1 \to 0 \text{ as } Y \to \infty \end{array}\right] \tag{3.8}$$

solving [3.2] - [3.7] with the boundary conditions (3.8), we have

$$T_o = e^{-py} \tag{3.9}$$

$$C_0 = e^{-scy} \tag{3.10}$$

$$u_0 = [A_1 e^{-R_3 y} - \frac{Gre^{-py}}{[p^2 - p-(M+\frac{1}{k})]} - \frac{Gme^{-scy}}{[sc^2 - sc - (M+\frac{1}{k})]} \tag{3.11}$$

$$T_1 = [A_2 e^{-R_1 y} + \frac{e^{-py}}{iw}] \tag{3.12}$$

$$C_1 = \left[A_3 e^{-R_2 y} - \frac{sc\, e^{-scy}}{iw}\right] \tag{3.13}$$

$$u_1 = [A_4 e^{-R_4 y} + \frac{A_1 R_3 e^{-R_3 y}}{[R_3^2 - R_3 - (M^2 + \frac{1}{k} iw)]}$$

$$- \frac{G\, r\, p\, e^{-py}}{[P^2 - p - (M + \frac{1}{k} + iw)] \times [p^2 - p - (M + \frac{1}{k})]}$$

$$- \frac{Gr\, sc\, e^{-scy}}{[(sc^2 - sc - (M + \frac{1}{k} + iw)][(sc^2 - sc - (M + \frac{1}{k})]}$$

$$- \frac{GmA_2 e^{-R_2 y}}{[R_1^2 - R_1 - (M + \frac{1}{k} + iw)]} - \frac{Gr\, e^{-py}}{iw[p^2 - p - (M + \frac{1}{k} + iw)]}$$

$$- \frac{GmA_3 e^{-R_2 Y}}{[R_2^2 - R_2 - (M + \frac{1}{k} + iw)]} + \frac{Gm\, e^{-scy}}{[sc^2 - sc - (M + \frac{1}{k} + iw)]}$$

$$A_1 = \left[\frac{Gr}{[(p^2 - p - (M + \frac{1}{k}))]} + \frac{Gm}{[(s_c^2 - sc - (M + \frac{1}{k})]} \right], \qquad (3.14)$$

$$R_3 = \left[\frac{1 + \sqrt{1 + 4(M + \frac{1}{k})}}{2} \right] \quad A_2 = (1 - \frac{1}{wi}), \; R_1 = \frac{(p + \sqrt{(p^2 + 4piw)}}{2} \; ;$$

$$A_3 = (1 + \frac{sc}{iw}), \; R_2 = \frac{sc + \sqrt{sc^2 + 4iw}}{2}$$

$$A_4 = \frac{Grp}{(p^2 - p - (M + \frac{1}{k} + iw)(p^2 - p - (M + \frac{1}{k})}$$

$$+ \frac{Grsc}{(sc^2 - sc - (M + \frac{1}{k} + iw)(sc^2 - sc - (M + \frac{1}{k})}$$

$$- \frac{A_1 R_3}{[R_3^2 - R_3 - (M^2 + \frac{1}{k} + iw)]} + \frac{GrA_2}{[R_1^2 - R_1 - (M + \frac{1}{k} + iw)]}$$

$$+ \frac{Gr}{iw[p^2 - p - (M + \frac{1}{k} + iw)]} + \frac{GmA_3}{(R_2{}^2 - R_2 - (M + \frac{1}{k} + iw)}$$

$$- \frac{Gmsc}{(sc^2 - sc - (M + \frac{1}{k} + iw)}$$

hence the expressions for velocity, temperature and concentration of mass are given by

$$u(Y,t) = A_1 e^{-R_3 Y} - \frac{Gre^{-py}}{[p^2 - p - (M + \frac{1}{k})]} - \frac{Gme^{-scy}}{[sc^2 - sc - (M + \frac{1}{k} p)]}) + \varepsilon\, e^{iwt}$$

$$A_4 e^{-R_4 Y} + \frac{A_1 R_3 e^{-R_3 Y}}{[(R_3^2 - R_3 - (M^2 + \frac{1}{k} + iw)]}$$

$$- \frac{Gr\, p\, e^{-PY}}{[p^2 - p - (M + \frac{1}{k} + iw)][p^2 - p - (M + \frac{1}{k})]}$$

$$- \frac{Gr\, sc\, e^{-scy}}{[(sc^2 - sc - (M + \frac{1}{k} + iw)]\,[(sc^2 - sc - (M + \frac{1}{k})]}$$

$$- \frac{Gr\, A_2\, e^{-R_2 Y}}{[R_1^2 - R_1 - (M + \frac{1}{K} + iw)]}$$

$$+\frac{Gr\, e^{-py}}{iw(p^2-p-(M+\frac{1}{k}+iw)}-\frac{Gm\, A_3 e^{-R_2 Y}}{R_2^2-R_2-\ (M+\frac{1}{k}+iw)}$$

$$+\frac{Gm\, e^{-scY} sc}{[\, sc^2-sc-\ (M+\frac{1}{k}+iw)\,]} \tag{3.15}$$

$$T(Y,t)=[\, e^{-pY}+\varepsilon\, e^{iwt}(A_2 e^{-R_1 Y}+\frac{e^{-PY}}{iw})\,] \tag{3.16}$$

$$C(Y,t)=[\, e^{-scy}+\varepsilon\, e^{iwt}\,(\, A_3 e^{-R_2 Y}-\frac{sce^{-scY}}{iw}\,)\,] \tag{3.17}$$

The expressions for transient velocity, temperature and concentration of mass for $wt=\frac{\pi}{2}$ are givern by

$$\left.\begin{aligned} u\,(\,Y,\frac{\pi}{2w}\,) &= [\, u_0\,(Y)+\varepsilon\, u_1\,(Y)\,] \\ T\,(\,Y,\frac{\pi}{2w}\,) &= \left[T_0\,(Y)+\varepsilon\ T_1\,(Y)\right] \\ C\,(\,Y,\frac{\pi}{2w}\,) &= [\, C_0\,(Y)+\varepsilon\, C_1\,(Y)\,] \end{aligned}\right] \tag{3.18}$$

Then skin friction is given by

$$T_o=\left(\frac{\partial u}{\partial y}\right)y=0$$

$$=\left(\frac{\partial\, u_0}{\partial\, y}+\varepsilon\, e^{iwt}\frac{\partial\, u_1}{\partial\, y}\right)y=0$$

$$=(\,-R_3A_1+\frac{PGr}{(p^2-p-(M+\frac{1}{k}))}+\frac{Gm\, sc}{(sc^2-sc-(M+\frac{1}{k}))}+\varepsilon\, e^{iwt}$$

$$(\,-R_4A_4-\frac{A_1R_3^2}{[\, R_3^2-R_3-(M^2+\frac{1}{k}+iw)\,]}+\frac{Gr\, p^2}{[\, p^2-p-(M+\frac{1}{k}+iw)\,]\times[\, p^2-p-(M+\frac{1}{k})\,]}$$

$$+\frac{Gr\, sc^2}{[\, sc^2-sc-(M+\frac{1}{k}+iw)\,]\,[\, sc^2-sc-(M+\frac{1}{k}\,)\,]}+\frac{Gr\, A_2\, R_1}{[\, R_1^2-R_1-(M+\frac{1}{k}+iw)\,]}$$

$$+\frac{Gr\, p}{iw\,[\, p^2-p-\ (\,M+\frac{1}{k}+iw)\ ]}+\frac{Gm\, A_3\, R_2}{[\, R_2^2-R_2-(M+\frac{1}{k}+iw)\,]}$$

$$-\ \frac{Gm\, sc^2}{[\, sc^2-sc-(M+\frac{1}{k}+iw)\,]} \tag{3.19}$$

The Nusselt number is given by

$$Nu = -\left(\frac{\partial t}{\partial y}\right) y = 0$$

$$= [\ p + \varepsilon\, e^{iwt} \,(A_2 R_1 + \frac{p}{iw})] \tag{3.20}$$

Cass (ii) Considering $f(t') = e^{-w't'}$ (3.21)

The equations (2.8) - (2.10) take the following forms.

$$\frac{\partial^2 u}{\partial y^2} + (\, i + \varepsilon\, e^{-wt}) \frac{\partial u}{\partial y} = \frac{\partial u}{\partial t} + (\, M + \frac{1}{k})u - GrT - GmC \tag{3.22}$$

$$\frac{\partial^2 T}{\partial y^2} + p\,(\,1 + \varepsilon\, e^{-wt}\,)\,\frac{\partial T}{\partial Y} = p\frac{\partial T}{\partial t} \tag{3.23}$$

$$\frac{\partial^2 c}{\partial Y^2} + sc\,(1 + \varepsilon\, e^{-wt})\,\frac{\partial c}{\partial Y} = sc\frac{\partial c}{\partial t} \tag{3.24}$$

The boundary conditions in this case are

$$T = 1 + \varepsilon\, e^{-wt},\ C = 1 + \varepsilon\, e^{-wt},\ u = 0 \quad at \quad y = 0 \tag{3.25}$$

$T \to 0$, $C \to 0$, $u \to 0$ as $y \to \infty$

For solving the above equations subject to the boundary conditions (3.25) we assume that

$$\left.\begin{aligned} u\,(y,t) &= u_0(y) + \varepsilon\, e^{-wt} u_1\,(y) \\ T\,(y,t) &= T_0(y) + \varepsilon\, e^{-wt} T_1\,(y) \\ C\,(y,t) &= C_0 Y + \varepsilon\, e^{-wt}\, C_1\,(y) \end{aligned}\right] \tag{3.26}$$

The expressions forvelocity, temperature and concentration of mass are given by

$$u\,(y,t) = [\,(\,R_4 e^{-R_5 Y}\,] - \frac{Gr\, e^{-py}}{[\,p^2 - p - (\,M + \frac{1}{k})\,]} - \frac{Gm\, e^{-scy}}{[\,sc^2 - sc - (M + \frac{1}{k})\,]} + \varepsilon\, e^{-wt}$$

$$R_7 e^{-R_6 Y} + \frac{R_4 R_5 e^{-R_5 y}}{(R_5^2 - R_5 - (M + \frac{1}{k} - iw)} - \frac{Gr\, pe^{-py}}{[\,p^2 - p - (M + \frac{1}{k})]\,[\,p^2 - p - (M + \frac{1}{k} - w)\,]}$$

$$- \frac{Gm\, Sce^{-Scy}}{[Sc^2 - Sc - (M + \frac{1}{K})\,]\,[Sc^2 - Sc - (M + \frac{1}{k} - w)\,]} - \frac{Gr\,(1 - p/w)\, e^{-k_1 y}}{[\,R_1^2 - R_1 - (\,M + \frac{1}{k} - w)\,]}$$

$$- \frac{Gr\, p\, e^{-py}}{w\,[\,p^2 - p - (M + \frac{1}{k} - w\,)\,]} - \frac{Gm\,(1 - sc/w) e^{-R_2 y}}{[\,R_2^2 - R_2 - (M + \frac{1}{k} - w)\,]}$$

$$- \frac{Gm\,(sc/w)\, e^{-scy}}{[\,(sc^2 - sc - (M + \frac{1}{k} - w)\,]}) \tag{3.27}$$

$$T(y,t) = e^{-py} + \varepsilon\, e^{-wt}\,[\,1 - \frac{p}{w} e^{-R_1 y} + \frac{p}{w} e^{-py}\,] \tag{3.28}$$

$$c(y,t) = e^{-scy} + \epsilon e^{-wt}\left[\left(1-\frac{sc}{w}\right)e^{-R_2 y} + \frac{sc}{w}e^{-scy}\right] \qquad (3.29)$$

where $R_1 = \dfrac{P+\sqrt{p^2-4pw}}{2}$, $R_2 = \dfrac{sc+\sqrt{sc^2-4wsc}}{2}$,

$$R_4 = \frac{Gr}{[p^2-p-(M+1/k)]} + \frac{Gm}{[sc^2-sc-(M+\frac{1}{k})]},$$

$$R_5 = \frac{1+\sqrt{1+4\left(M+\frac{1}{k}\right)}}{2}, \quad R_6 = \frac{1+\sqrt{1+4\left(M+\frac{1}{k}-w\right)}}{2}$$

$$R_7 = \frac{Gr\,p}{[p^2-p-(M+\frac{1}{k})][p^2-p-(M+\frac{1}{k}-w)]} - \frac{R_4R_5}{[R_5^2-R_5-(M+\frac{1}{k}-w)]}$$

$$+\frac{Gm\,sc}{[sc^2-sc-(M+\frac{1}{k})\times[sc^2-sc-(M+\frac{1}{k}-w)]} + \frac{Gr(1-p/w)}{[R_1^2-R_1-(M+\frac{1}{k}-w)]}$$

$$+\frac{Gr(p/w)}{[p^2-p-(M+\frac{1}{k}-w)]} + \frac{Gm(1-sc/w)}{[R_2^2-R_2-(M+\frac{1}{k}-w)]} + \frac{Gm(sc/w)}{[sc^2-sc-(M+\frac{1}{k}-w)]}.$$

4. Discussion of results

We have considered here the effects of various parameters onvelocity and temperature

Table 1: Velocity Distribution for Different Values of Gr

y	t	Gr	u
0	1	2.5	17.899172
0	2	2.5	17.898935
0	3	2.5	17.898747
1	1	2.5	-6.9115447
1	2	2.5	-6.9160344
1	3	2.5	-6.9204315
0	1	4.00	-22.554888
0	2	4.00	-22.54611
0	3	4.00	-22.536789
1	1	4.00	-5.0162558
1	2	4.00	-5.0339889
1	3	4.00	-5.0513563
0	1	10.00	-27.428088
0	2	10.00	-33.48559
0	3	10.00	-33.546648
1	1	10.00	-15.087494
1	2	10.00	-15.073846
1	3	10.00	-15.059355

In the above calculations we have taken

Sc= o.81, P=o.71, W=0.05, ε=.005, M= 0.5 Gm=1,K=2.

for case two.

Table 2: Effect of Gm On Velocity Profile

y	t	Gm	u
0	1	1.00	-25.010407
0	2	1.00	-25.01046
0	3	1.00	-25.010515
1	1	1.00	-12.31247
1	2	1.00	-12.318144
1	3	1.00	-12.324168
0	1	3.00	-3.4744953
0	2	3.00	-3.4720575
0	3	3.00	-3.4694689
1	1	3.00	-.7369553
1	2	3.00	-.7415056
1	3	3.00	-.746336
0	1	5.00	-1.2656925
0	2	5.00	-1.2912297
0	3	5.00	-1.3183547
1	1	5.00	-1.990067
1	2	5.00	-1.9962003
1	3	5.00	-2.002713

Here we have considered p=0.71, ε=.005, Gr=2.5, Sc=.81, W=.05, K=2, M=0.5.

Table 3: Velocity Distribution for Different Values of M

y	t	M	u
0	1	1.00	-.1759285
0	2	1.00	-.1599099
0	3	1.00	-.1442216
1	1	1.00	-.4246094
1	2	1.00	-.4240426
1	3	1.00	-.4232382
0	1	1.25	-.0241149
0	2	1.25	-.0211322
0	3	1.25	-.0182113
1	1	1.25	.4176628
1	2	1.25	.4164999
1	3	1.25	.415361
0	1	1.50	-1.419598
0	2	1.50	-1.4227777
0	3	1.50	-1.4258918
1	1	1.50	-.2434641
1	2	1.50	-.243751
1	3	1.50	-.2440319

In the above calculations we have taken

Sc=0.81, w= .05 ε-.005, Gr=2.5, Gm=1, P=0.71, K=2.

Table 4: Effect of Sc on Temperature Distribution

y	t	Sc	T
0.4	1	.16	.7595321
0.4	2	.16	.7594582
0.4	3	.16	.7593798
1.0	1	.16	.4990338
1.0	2	.16	.4989345
1.0	3	.16	.4989345
0.4	1	.60	.7579699
0.4	2	.60	.757927
0.4	3	.60	.7578814
1.0	1	.60	.4956812
1.0	2	.60	.4875994
1.0	3	.60	.4957187
0.4	1	.81	.7585143
0.4	2	-81	.7584606
0.4	3	.81	.7584036
1.0	1	.81	.4990581
1.0	2	.81	.4990095
1.0	3	.81	.4989578
0.0	1	1.50	.7579774
0.0	2	1.50	.7578853
0.0	3	1.50	.7578406
1.0	1	1.50	.4972714
1.0	2	1.50	.4972589
1.0	3	1.50	.4973535

Here we have considered p= 0.71, ε=.005, K=2, W=.05, Gm=1,Gr=2.5, M=.5.

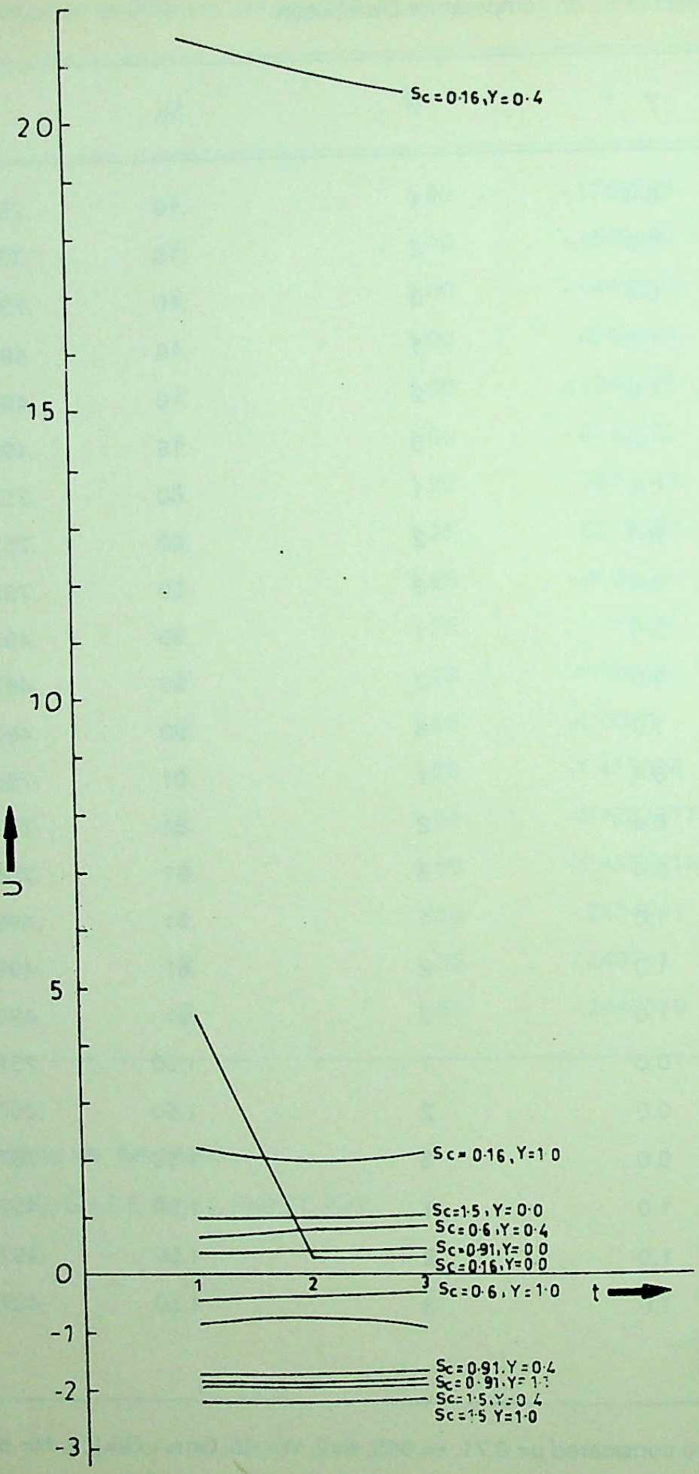

Fig. 1: Effect of Sc on velocity profile

From the above mumerical calculations it can be said that different parameters have different influence on velocity and temperature profiles.

In figure 1 the effect of Sc on the velocity profile has been shown at different times.

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Common fixed point theorems for finite number of mappings under a weak commutativity condition.

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Abstract. We have considered finite number of self maps on a metric space which are weakly commutating in the space and then we have found that these maps have unique fixed points under suitable conditions.

1. Introduction

Suppose A_1, A_2 is a pair of self maps over a metric space (X,d) satisfying a weaker condition than (Fisher and Sessa, 1990) commutativity namely

$$d(A_1A_2xA_2A_1x)\ \ d(A_1x, A_2x) \qquad (1.1)$$

for each x, $\in X$. Two commuting maps satisfy the condition (1.1) but the converse is not true. We give an example to explain that weak commutativity does not imply commutativeness of two functions.

Example 1 Let X = [0,1] with euclidean metric and let us take

$$A_1x = \frac{x}{x+10}, \quad A_2x = \frac{x}{5} \quad \text{for each } x \in X.$$

$$\text{Then } d(A_1A_2x, A_2A_1x) = \frac{x}{x+50} - \frac{x}{5x+50} = \frac{4x^2}{(x+50)(5x+50)}$$

$$\text{and } d(A_1x, A_2x) = \frac{x}{5} - \frac{x}{x+10} = \frac{x^2+10x-5x}{5(x+10)}$$

$$= \frac{x^2+5x}{5(x+10)} = \frac{x(x+5)}{5(x+10)}$$

So, we find $4x^2 \le x(x+5)(x+50)$

i.e. $d(A_1A_2x, A_2A_1x) \le d(A_1x, A_2x)$.

Moreover A_1 and A_2 do not commute.

Here we follow the notations of Rhoades and Sessa (1990) namely

$F = \{f: R^+ \to R^+\}$ f is non-decreasing, right continuous}

and satisfies (i) $f(t) < t$ for every $t > O$

(ii) $t - f(t) \to \infty$ as $t \to \infty$.

We establish some common fixed point theorems for a finite number of self maps satisfying the following conditions:

For any $x \in X$, there exists a $y \in X$ such that

$A_1x = A_2y = A_3y = \ldots\ldots\ldots\ldots = A_n y$ (1-2)

$d(A_1x, A_1y) \leq f(\max\{d(A_2x, A_3y),\ d(A_2y, A_3x)$ (1-3)

$d(A_nx, A_1y),\ d(A_1x, A_ny)\}$,

for every $x, y \in X$ and $f \in F$

Our result generalizes the result of Rhoades and Sessa (1990)

Here we state (without proof) three lemmas and few notations borrowed from Singh and Meade (1982), Rhoades and Sessa (1990):

δ (A) denotes the diameter of a set A ; $O(y_k, n) = \{y_k, \ldots\ldots\ldots\ldots y_{k+n}\}$,

$k = 0, 1, 2, \ldots\ldots\ldots\ldots$;

$O(y_o, \infty) = \{y_o, y_1, y_2, \ldots\ldots\ldots\ldots\}$.

Lemma 1 If $t > O$, then $f^n(t) \to 0$ as $n \to \infty$

Lemma 2 Suppose $\delta\ (O(y_k, n)) > 0$ for some $k \geq 0,\ n \geq 1$

Then $1 = \delta\ (O(y_o, \propto)) < \propto$ and $\cdot\ \delta \cdot (O(y_k, n) \geq f^k(1)$, for each $k \geq 0,\ n \geq 1$

Lemma 3 : Suppose $\delta\ (O(y_k, n)\ 0$ for some $k \geq 0,\ n \geq 1$.

Then $\{y_n\}$ is a Cauchy Sequence.

We state our theorems

Theorem 1 Let $A_1, A_2, A_3, \ldots\ldots\ldots A_n$ be n-self mappings of (X, d) satisfying (1.2) and

$$d(A_1x, A_1y) \geq f(\max\{d(A_2x, A_3x), d(A_2y, A_3y), d(A_2x, A_3y), d(A_2y, A_3x),\ d(A_1x, A_2x),\ d(A_1y, A_2y),\ d(A_1x, A_2y)\ d(A_1y, A_2x), \ldots\ldots\ldots\ldots\ d(A_1x, A_nx), d(A_1y, A_ny), d(A_1x, A_ny),\ d(A_1y, A_nx)\}) \quad (2.1)$$

for every $x, y \in X$ and $f \in F$.

If $(A_1, A_2), (A_1, A_3), \ldots\ldots\ldots (A_1, A_n)$ are weakly commuting pairs and $\delta \ (0(y_k,n) = 0$ for some $k \geq 0$, $n \geq 1$, then $A_1,A_2,......A_n$ have a unique common fixed point.

Proof from the hypothesis, $y_k = y_{k+1}$ and so $A_1x_{k+1} = A_2x_{k+1} = ... = A_nx_{k+1}$. For convenience, let us denote $x_{k+1} = p$.

Then

$$A_1p = A_2p = ... = A_np \tag{2.2}$$

and by (1.1) we have

$$d(A_1A_2p,A_2A_1p) \leq d(A_2p,A_1p) = 0 \tag{2.3}$$

$$d(A_1A_np,A_nA_1p) \leq d(A_np,A_1p) = 0$$

Applying (2.2) and (2.3), we get

$$A_1A_1p = A_1A_2p = A_2A_1p = .. A_1A_np = A_nA_1p \tag{2.4}$$

Again by (2.1)

$d(A_1(A_1p),A_1p) \leq f\,(\max\{d(A_1(A_1p),\ A_2(A_1p),d(A_1p,A_2p),$

$d(A_1(A_1p),A_2p),d(A_2(A_1p),A_1p),\ d(A_2(A_1p),$

$A_3(A_1p)),\ d(A_2p,A_3p),\ d(A_2(A_1p),A_3p),$

$d(A_2p,A_3(A_1p),..\ d(A_1(A_1p),A_n(A_1p)),$

$d(A_1p,A_np),d(A_1(A_np),A_np)),d(A_1p,A_n(A_1p))\}).$

$= f\,(\max\{0,0,\ d(A_1(A_1p),A_2p),\ d(A_2A_1p,A_1p),\ ..\ 0,0,$

$d(A_1(A_1p),A_np),d(A_1p,A_nA_1p)\}).$

$= f\,(\max\{d(A_1(A_1p),A_1p),..d(A_1A_1p,A_1p)\}).$

From (i), $A_1A_1p = A_1p = A_1p$ is a fixed point of A_1 and therefore also for $A_2,A_3,..,A_n$, from (2.4)

Taking $A_1p = z$ and let z' be another common fixed point of $A_1,A_2,\ .\ .\ .\ .\ .\ .\ A_n$.

Then $d(z,z') = d(A_1z,A_1z')$

$f\,(\max\{d(A_1z,A_2z),\ d(z,A_2z'),d(A_1z,A_2z'),\ d(A_2z,z')..,$

$.\ .\ .\ .\ .,\ d(Anz,A_1z),d)A_nz',A_1z'),d)A_nz',A_1z),d(A_nz,A_1z')\}).$

$= f(d(z,z'))$

$\Rightarrow z=z'$, otherwise $d(z,z') \leq f\,(d(z,z') < d\,(z,z')$, a contradiction.

Corollary If we take $n = 3$ and $A_1 = A$, $A_2 = S$ and $A_3 = T$, our theorem becomes theorem 1 of (1990)

Theorem 2 Let $\{A_i\}^{n}{}_i = 1$ be n-self mappings of a complete metric space (X,d) satisfying (1.2) and (2.1) as in theorem 1 and let $\delta(O(y_k,n)) > 0$ for some $k \ge 0$, $n \ge 1$ and $\{A1, Ai \mid 1 < i \le n\}$ are weakly commuting pairs. If

(a_1) either A_i is continuous and $d(x,A_jx) \le d(x,A_ix)$ for any $x \in X$, or

(a_2) A_j is continuous and $d(x,A_ix) \le d(x,A_jx)$ for any $x \in X$, $1 < i \ne j \le n$.

(a_3) A_1 is continous,

then A_1, A_i, A_j have a unique common fixed point.

Proof Let (a_1) be true. Using lemma 3 and completeness of X, the sequence $\{y_n\} = \{A_1x_n\} = \{A_2x_{n+1}\} = .. \{A_nx_{n+1}\}$ converges to a point p. Then the sequence $\{A_i^2 x_{n+1}\} = \{A_iA_1x_n\}$ converges to A_ip, $1 < i \le n$. Therefore,

$$\begin{aligned} d(A_1A_ix_n, A_ip) &\le d(A_1A_ix_n, A_iA_1x_n) + d(A_iA_1X_n, A_1p) \\ &\le d(A_ix_n, A_1x_n) + d(A_iA_1x_n, A_ip) \end{aligned}$$

Which implies that, as $n \to \propto$, the sequence $\{A_1A_ix_n\}$ converges to A_ip.

Since $\{A_1, A_j\}$, $1 < j \le n$, is a weakly commuting pair,

$$\begin{aligned} d(A_jA_ix_n, A_ip) &= d(A_jA_1x_{n-1}, A_1p) \\ &\le d(A_jA_1X_{n-1}, A_1A_jx_{n-1}) + d(A_1A_jx_{n-1}, A_ip) \\ &\le d(A_jX_{n-1}, A_jx_{n-1}) + d(A_1A_jx_{n-1}, A_ip) \end{aligned}$$

Now taking $n \to \propto$, the sequence $\{A_jA_ix_n\}$ converges to A_ip.

Using (2.1)

$$\begin{aligned} d(A_1A_ix_n, A_1x_n) \le f\,(\max\{&d(A_i^2x_n, A_jx_n), d(A_jA_ix_n, A_ix_n), \\ &d(A_i^2x_n, A_ix_n), d(A_jA_ix_n, A_ix_n), \\ &d(A_jA_ix_n, A_1A_ix), d(A_ix_n, A_1x_n), .. \\ &d(A^2_i x_n, A_1x_n), d(A_ix_n, A_1A_ix_n), .. \\ &d(A_jA_ix_n, A_1x_n), d(A_jx_n, A_1A_ix_n)\}). \end{aligned}$$

As $n \to \propto$ and f being right continuous,

$$\begin{aligned} d(A_ip, p) \le\ & f\,(\max\{d(A_ip,p), d(A_ip,p), d(A_ip,A_ip), d(p,p), \\ & d(A_ip,A_ip), d(p,p), d(A_ip,p)\ d(p,A_ip), d(A_ip,p), d(p,A_ip)\}). \\ =\ & f\,(\max\{d(A_ip,p), d(A_ip,p), d(A_ip,p), d(p,A_ip), \\ & d(A_ip,p), d(p,A_ip)\}) \end{aligned}$$

$= f(d(A_ip,p))$.

$\Rightarrow$ $A_ip = p$; Similarly $A_jp = p$.

Using 2.1

$d(A_1p,A_1x_n) \leq f(\max\{d(A_ip,A_j x_n), d(A_jp,A_i x_n), d(A_ip,A_1p),$

$d(A_ix_n,A_1x_n), d(A_jp,A_1p), d(A_jx_n,A_1x_n),$

$d(A_ip,A_1x_n), d(A_ix_n,A_1p), d(A_jp,A_1x_n),$

$d(A_jx_n, A_1p)\})$.

As $n \to \propto$ and f being right continuous,

$$\begin{aligned} d(A1p,p) &\leq f(\max\{d(A_ip,p), d(A_jp,p), d(A_ip,A_1p), d(p,p), d(A_jp,A_1p), \\ &\qquad d(p,p), d(A_ip,p), d(p,A_1p), d(A_jp,p), d(p,A_1p)\}). \\ &= f(\max\{d(p,p), d(p,p), d(p,A_1p), d(p,p), d(p, A_1p), d(p,p), \\ &\qquad d(p,p), d(p,A_1p), d(p,p), d(p,A_1p)\}). \\ &= f(d(p,A_1p)) \end{aligned}$$

$\Rightarrow p = A_1p$. Therefore $p = A_1p = A_jp = A_ip$.

Similar argument applies for (a_2). Suppose (a_3) holds.

Then the sequence $(A_1^2x_n\} = \{A_1A_ix_{n+1}\} = \{A_1A_jxn_{+1}\}$ converges to A_1p.

By (1.1), $d(A_iA_1x_n,A_1p) \leq d(A_iA_1x_n, A_1A_ix_n) + d(A_1A_ix_n,A_1p)$

$\leq d(A_1x_n, A_ix_n) + d(A_1A_ix_n, A_1p)$.

$\Rightarrow \{A_iA_1x_n\}$ converges to A_1p. Similarly $\{A_jA_1x_n\}$ converges to A_1p. Also using (2.1)

$$\begin{aligned} d(A_1^2x_n,A_1x_n) \leq\ & f(\max\{d(A_iA_1x_n, A_jx_n), d(A_jA_1x_n), d(A_iA_1x_n,A_1^2x_n), \\ & d(A_ix_n, A_1x_n), d(A_jA_1x_n, A_1^2x_n), d(A_jx_n,A_1x_n), \\ & d(A_iA_1x_n,A_1x_n), d(A_ix_n A_1^2x_n), d(A_jA_1x_n,A_1x_n), \\ & d(A_jx_n,A_1^2x_n\}). \end{aligned}$$

As $n \to \propto$ and f being right continuous, we get

$d(A_1p,p) \leq f(d(A_1p,p)) \Rightarrow p = A_1p$, by (i).

From (1.2) there exists a point $u \in X$ such that

$p = A_1p = A_iu = A_ju$.

Again, using (2.1),

$$\begin{aligned} d(A_1x_n,A_1u) \leq\ & f(\max\{d(A_ix_n,A_ju), d(A_jx_n, A_iu), d(A_ix_n, A_1x_n), \\ & d(A_iu, A_1u), d(A_jx_n,A_1x_n), d(A_ju,A_1u), d(A_ix_n,A_1u), \\ & d(Ai_u,A_1x_n), d(A_jx_n,A_1u), d(A_ju,A_1x_u)\})). \end{aligned} \tag{2.5}$$

As $n \to \alpha$ and f being right continuous, we get

$$d(p,A_1u) \le f(\max\{d(p,A_ju), d(p,A_iu), d(p,p), d(A_iu,A_1u), d(p,p), d(A_ju,A_1u), d(p,A_1u), d(A_iu,p), d(p,A_1u), d(A_ju,p)\}).$$

$$= f(d(p,A_1u)).$$

$\therefore$ $p = A_1p = A_1u \ne A_iu = A_ju$. (2.6)

Hence $p = A_1p = A_1A_iu = A_iA_1u = A_ip$ and

$p = A_1p = A_1A_ju = A_jA_1u = A_jp$.

Therefore, p is a common fixed point of A_1, A_i, A_j.

- For uniqueness, suppose p^* is another common fixed point of $A_1, A_2, .. A_n$.

Then $d(p,p^*) = d(A_1p, A_1p^*)$

$$\le f(\max\{d(A_1p,A_2p), d(A_1p^*,A_2p^*), d(A_1p^*,A_2p), d(A_1p,A_2p^*) .. d(A_np,A_1p^*)\})$$

$$\le f(\max\{d(A_1p,A_np), d(A_1p^*,A_np^*), d(A_1p^*,A_np), d(A_1p,A_np^*)\})$$

$$= f(d(p,p^*)).$$

$\Rightarrow p = p^*$, therefore, A_1, A_i, A_j have a unique common fixed point p.

Example 2 Consider $X = [0,1]$ and $A_1x = \frac{x}{x+4\alpha}$, $\alpha > \frac{n-2}{4}$,

$$A_nx = \frac{x}{4}, \quad A_ix = \frac{x}{4}, \; x \in [0, \tfrac{1}{i}] \qquad i = 2,3,..n-1.$$

$$x \in [\tfrac{1}{i}, 1]$$

We find A_1, A_n is weakly commutative because of the example (1) and whenever $x \in [0, \frac{1}{i}]$, by the example (1), A_1 and A_i are weakly commutative. Now, when $x \in [\frac{1}{i}, 1]$,

$A_iA_1x = A_i\left(\frac{x}{x+4\alpha}\right) = \frac{x}{4(x+4\alpha)}$, Since $\frac{x}{(x+4\alpha)} < \frac{1}{i}$.

and $A_1A_ix = A_1(a) = \frac{1}{1+4\alpha}$, Therefore,

$$d(A_1A_ix, A_iA_1x) = \frac{4\alpha x - 3x - 16\alpha}{4(1+4\alpha)(x+4\alpha)} \le \frac{4\alpha}{x+4\alpha}$$

$$= d(A_1x, A_ix),\ i = 2,3,\ldots, \quad n-1.$$

This means that A_1, A_i are weakly commutative for every x in X.

Now $A_1X = [0, \frac{1}{4\alpha+1}]$, and so, we assume $y = 4A_1(x)$ for any x in X in order to verify(1.2)

Furthermore, (2.1) is also satisfied. Then we find $A_1, A_2, .. A_n$ have a unique common fixed point 0.

Remark : From the above example, we find, $A_1x = [0, \frac{1}{1+4\alpha}]$,

$A_1x = \frac{x}{x+4\alpha}$ and $y = 4A_1(x)$,

$$A_iy = A_i\left(\frac{4x}{x+6}\right) = \frac{x}{x+6}$$

$$d(A_1x, A_1y) = \frac{x}{x+4\alpha} - \frac{y}{y+4\alpha} = \frac{xy+4\alpha x - xy - 4\alpha y}{(x+4\alpha)(y+4\alpha)}$$

$$= \frac{4\alpha(x-y)}{(x+4\alpha)(y+4\alpha)}$$

$$d(A_1x, A_ny) = \frac{x}{x+4\alpha} - \frac{y}{4} = \frac{4x - xy - 4\alpha}{4(x+4\alpha)}$$

$$d(A_ix, A_ny) = \frac{x}{4} - \frac{y}{4} = \frac{x-y}{4}, \quad x \in [0, \tfrac{1}{i}]$$

$$\therefore d(A_ny, A_ix) = 1 - \frac{y}{4} + \frac{4-y}{4}, \quad x \in [\tfrac{1}{i}, 1]$$

$$d(A_ix, A_1y) = \frac{x}{4} - \frac{y}{y+4\alpha} = \frac{xy + 4\alpha x - 4y}{4(y+4\alpha)}, \quad x \in [0, \tfrac{1}{i}]$$

$$= 1 - \frac{y}{y+4\alpha} = \frac{4\alpha}{y+4\alpha}, \quad x \in [\tfrac{1}{i}, 1]$$

$$d(A_ix, A_iy) = \frac{x-y}{4}, \quad x, y \in [0, \tfrac{1}{i}]$$

$$d(A_ix, A_iy) = \frac{1-x}{4}, \quad x \in [o, \tfrac{1}{i}], \; y \in [\tfrac{1}{i}, 1]$$

$=0$, $y, x \in [\frac{1}{i}, 1]$.

Condition (a_3) of theorem 2 is satisfied since A_1 is continuous.

An is continous but (a_1) is not satisfied since

$$d(x, A_ix) > d(x, A_nx), \quad x \in [\tfrac{1}{i}, \tfrac{4}{7}].$$

Also (a_2) is not satisfied because A_i's are not continuous at $\frac{1}{i}$, i = 2, 3,.., n-1. 0 is the unique common fixed point of all $A_i^{'s}$, i = 1,2,..,n, but Ai, i = 2,3,..,n-1, has 0 and 1 as common fixed points. Thus conditions (a_1) and (a_2) are independent of each other. Now we state our last theorem which can be proved in similar fashion.

Theorem 3 Let A_1, A_2, A_3, ..A_n be n-self mappings of a complete metric space (X,d) satisfying (1.2) and

$$d(A1x,A1y) \le f(\max \le \{d(A_2x,A_3y), d(A_2x, A_1x), d(A_2y,A_1y), d(A_2y,A_1y), d(A_3y,A_1x),..d(A_1x,A_ny), d(A_1y,A_ny)\}).$$

for any x, y $\in$ X, where f $\in$ F. If {A_1, A_3}, {A_1,A_2},..{A_1,A_n} are weakly commuting pairs and one of A_1, A_2, A_3,.. A_n is continuous then A_1,A_2,A_3,..A_n have a unique common fixed point.

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Some results of the functions related to the set C_k

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Abstract. In this paper,we shall construct functions f(x) and v(x) related to Cantor type sets C $_k$ for a given positive integer k and study some properties of these functions.

1. Introduction

In our earlier paper (1992) we have constructed Cantor type sets C_k for a given positive integer k. the set

$$C_k = x \in [0,1]/x = \sum_{k=1}^{\infty} \frac{x_i}{(2k+1)^i} = x_i = (0,2,4,\ldots,2k)\ \forall_i \}$$

We have shown that the set C_k is a non-dense perfect set and aslo Lebesque measure of C_k is zero. We now define two functions f (x) and v (x) as follows :

If x is any point in $0 \leq x \leq 1$,, then we can express x in the form

$$x = \sum_{i=1}^{\infty} \frac{x_i}{(2k+1)^i},$$ where $x_i = (o, 1, 2, \ldots, 2k)$ for every i. we define

$$f(x) = \sum_{i=1}^{\infty} \frac{1}{(2k+1)^i} \quad \text{and} \quad v(x) = \sum_{i=1}^{\infty} \frac{v_i(x)}{(2k+1)^i} \quad where\ \ f_i(x) = 2\delta(x_{i.}\ 2n)$$

and $v_i(x) = 2\delta\,(x_{1.}\ 2n-1)$, $n = 1,2,\ldots,k$ satisfying the relations

$$\begin{aligned}\delta\,(x_i, 2n) &= n, \text{ if } x_i = 2n\\ &= 0, \text{ if } x_i \neq 2n\\ (x_i, 2n-1) &= n, \text{ if } x_i = 2n-1\\ &= 0, \text{ if } x_i \neq 2n-1\end{aligned}$$

it follows that $f(x) \in C_k$ and $v(x) \in C_k$. We shall now establish some properties of the function f (x) in connection with the set C_k. The same properties hold good for the function v (x) also.

In our paper (1992) we have genralised Randolph's (1940) property related to the set C_k. We have shown that given two positive real numbers μ and ν satisfying $\frac{1}{2k+1} \leq \frac{\mu}{\nu} \leq 1$, eachpoint α in $o \leq \alpha \leq 1$ divides a segment $[x, y] \leq [0, 1]$ in the ratio $\nu : \mu$ whose end points x and y are the points of C $_k$.

We shall now prove the following therorems.

Theorem 1. Given two positive real numbers μ and ν with $\frac{1}{2k+1} \le \frac{\mu}{\upsilon} \le 1$ and any point $d \in [0, 1]$ the cardinality of the set of points $\{(x, y), 0 \le x, y \le 1$ satisfying $\alpha = \frac{\mu f(y) + \nu f(x)}{\mu + \nu}$ is either finite or c where f is the function defined above.

Proof : Let x be any point in [0, 1]. Then we write

$$x = \sum_{i=1}^{\infty} \frac{x_i}{(2k+1)^i} \quad x_i = (o, 1, 2, \dots, 2k) \text{ for all } i$$

we shall express x uniquely by replacing any final '1' in it with a chain of 2 k' s. We take

$$f(x) = \sum_{i=1}^{\infty} \frac{f_i(x)}{(2k+1)^i} \text{ and } v(x) = \sum_{i=1}^{\infty} \frac{v_i(x)}{(2k+1)^i}$$

where $f_i(x) = 2\delta(x_i, 2n)$ and $v_i(x) = 2\delta(x_i, 2n-1)$ with the the conditions

$\delta(x_i, 2n) = n$, if $x_1 = 2n$ $(n = 1, 2, \dots, k)$

$\delta(x_i, 2n) = o$, if $x_i \ne 2n$

and $\delta(x_i, 2n-1) = n$, if $x_i = 2n-1$

$\delta(x_i, 2n-1) = 0$, if $x_i \ne 2n-1$.

so $f_i(x) = (0,2,4,\dots,2k)$ and $v_i(x) = (0,2,4,\dots,2k)$ for every i and hence $f(x) \in C_k$ and $v(x) \in C_k$.

In our paper written jointly with Ganguly and Ray (1989) we have proved that f and v map the unit interval [0, 1] on to the Cantor type set C_k and except for an enumerable subset of C_k, every point of C_k is the image of continuum number of points $x \in [0, 1]$ under the mapping by each of the functions f and v.

In our earlier paper (1992) we have shown that for each point d in [0, 1] there corresponds a segment $[\beta, \alpha] \le [0, 1]$ such theat $d = \frac{\mu\beta + \nu\alpha}{\mu + \nu}$ with $\beta \in C_k$ and $\alpha \in C_k$ and for any two positive real numbers μ and ν such that $\frac{1}{2k+i} \le \frac{\mu}{\upsilon} \le 1$. for these fixed α, β we consider the sets

$$E = [x \in (0,1)/fx = \alpha] \text{ and } F = [\gamma \in (0,1)/f(\gamma) = \beta]$$

Then $d = \frac{\mu\beta + \upsilon\alpha}{\mu + \upsilon} = \frac{\mu f(\gamma) + \upsilon f(x)}{\mu + \upsilon}$ for any $X \in E$ and $Y \in F$. Since the cardinality of the sets E and F are either finite or c, it follows that the cardinality of the set EXF is either finite or C^2 (= C) (Hobson 1927). Hence the power of the set {(x, y)} corresponding to a given *$d \in [0,1]$ and a number $\frac{\mu}{\upsilon}$, with $\frac{1}{2k+1} \le \frac{\mu}{\upsilon} < 1$ satisfying* $\frac{\mu f(y) + \upsilon f(x)}{\mu + \upsilon}$ is either finite or C.

Then $d = \frac{\mu\beta + \upsilon\alpha}{\mu + \upsilon} = \frac{\mu f(\gamma) + \upsilon f(x)}{\mu + \upsilon}$ for any $X \in E$ and $Y \in F$. Since the cardinality of the sets E and F are either finite or c, it follows that the cardinality of the set EXF is either finite or C^2 (= C) (Hobson 1927). Hence the power of the set $\{(x, y)\}$ corresponding to a given $d \in [0,1]$ and *a number* $\frac{\mu}{\upsilon}$, *with* $\frac{1}{2k+1} \leq \frac{\mu}{\upsilon} < 1$ *satisfying* $\frac{\mu f(y) + \upsilon f(x)}{\mu + \upsilon}$ is either finite or C.

Note : Similar result can be proved for the function v.

Theorem 2. The function f is midway convex on the set C_k i.e.

$f\left(\frac{x_1 + x_2}{2}\right) < \frac{f(x_1) + f(x_2)}{2}$ for any two points x_1 and x_2 belonging to C_k.

In order to prove the result we first prove a lemma.

Lemma : For any $x \in C_k$, $f(x) = x$.

Proof of the lemma : As $x \in C_k$, x can be expressed as

$x = \sum_{i=1}^{\infty} \frac{a_i}{(2k+1)^i}$ where a_i = (0, 1, 2, ... 2k) for every i = 1, 2, 3, ...

Then $f(x) = \sum_{i=1}^{\infty} \frac{f_i(x)}{(2k+1)^i}$ *where* $f_i(x) = 2\delta(a_i, 2\lambda)$, $(\lambda = 1,2,\ldots,k)$

with $\delta(a_i, 2\lambda) = \lambda$, if $a_i = 2\lambda$.

$(q_i, 2\lambda) = 0$ if $a_i \neq 2\lambda$.

Now, if $a_i = 0$, $f_i(x) = 0$

if $a_i = 2$, $f_i(x) = 2$

if $a_i = 4$, $f_i(x)$ 4

if $a_i = 2k$, $f_i(x) = 2k$ for *every* $i = 1,2,3,\ldots$

$\therefore f_1(x) = a_i$

Thus, $f(x) = \sum_{i=1}^{\infty} \frac{f_i(x)}{(2k+1)^i} = \sum_{i=1}^{\infty} \frac{a_i}{(2k+1)^i} = x$

Hence every $x \in C_k$ satisfies the equation f (x) = x.

Proof of the theorem : Lat x_1 and x_2 be any two points of C_k.

Then $f(x_1) = x_1 = \sum_{i=1}^{\infty} \frac{x_i}{(2k+1)^i}$, $x_i = (0,2,4,\ldots 2k)$ $\forall$ i

and $f(x_2) = x_2 = \sum_{i=1}^{\infty} \frac{x_i}{(2k+1)^i}$, $x_i = (0,2,4,\ldots,2k)$ $\forall$ i

Hence

$$(i)\ldots\frac{f(x_1)+f(x_2)}{2} = \frac{x_1+x_2}{2} = \sum_{i=1}^{\infty} \frac{\frac{x_i+x_i}{2}}{(2k+1)^i} = \sum_{i=1}^{\infty} \frac{c_i}{(2k+1)^i}$$

Where $C_i = \frac{x_i + x_i'}{2}$

We now write, $x = \frac{x_1+x_2}{2}$ ....(1)

Where $x_1 \in C_k,\ \ x_2 \in C_k.$

Then

$$f\ \frac{x_1+x_2}{2} = f(x) = \sum_{i=1}^{\infty} \frac{f_i(x)}{(2k+1)^i} \ \ldots\ldots\ldots\ldots\ldots \ \ldots\ldots \quad (2)$$

$$\text{Where } x = \frac{x_1+x_2}{2} = \sum_{i=1}^{\infty} \frac{\frac{x_i+x_i'}{(2k+1)^i}}{2}$$

$$= \sum_{i=1}^{\infty} \frac{\frac{x_i+x_i'}{2}}{(2k+1)^i} = \sum_{i=1}^{\infty} \frac{C_i}{(2k+1)^i} \text{ by (1)}$$

if $\begin{cases} x_i = 0 \\ x_i' = 0 \end{cases}$

$C_i = 0$ and thus $f_i(x) = 0 = C_i$

if $\begin{cases} x_i = 0 \\ x_i' = 2 \end{cases}$

$C_1 = 1$ and thus $f_i(x) = 0 < C_i$

jf $\begin{cases} x_i = 0 \\ x_i' = 4 \end{cases}$

$C_i = 2$ and thus $f_i(x) = 2 = C_i$

if $\begin{cases} x_i = 0 \\ x_i' = 6 \end{cases}$

$C_i = 3$ and thus $f_i(x) = 0 < C_i$

if $\begin{cases} x_i = 0 \\ x_i' = 8 \end{cases}$

$C_i = 4$ and thus $f_i(x) = 4 = C_i$

....

if $\begin{cases} x_i = 0 \\ x_i' = 2k \end{cases}$

and thus $f_i(x) = 0$ if $k = 1,3,5,\ldots 2k-1$;

$f_i(x) = k$, if $k = 2,4,6,\ldots 2k$;

and thus $f_1(x) \le C_1$

Again if

if $\begin{cases} x_i = 2 \\ x_i' = 0 \end{cases}$

$C_i = 1,\quad f_i(x) = 0 < C_i$

if $\begin{cases} x_i = 4 \\ x_i' = 0 \end{cases}$

$C_i = 2,\quad f_i(x) = 2 = C_i$

if $\begin{cases} x_i = 6 \\ x_i' = 0 \end{cases}$

$C_i = 3,\quad f_i(x) = 0 < C_i$

if $\begin{cases} x_i = 8 \\ x_i' = 0 \end{cases}$

$C_i = 4,\quad f_i(x) = 4 = C_i$

……

if $\begin{cases} x_i = 2k \\ x_i' = 0 \end{cases}$

$C_i = k$, and thus $f_i(x) = 0$, if $k = 1,3,5,\ldots 2k-1$

$= k$, if $k = 2,4,6,\ldots 2k$

and thus $f_i(x) \le C_i$

if $\begin{cases} x_i = 2 \\ x_i' = 2 \end{cases}$

$C_i = 2,\quad f_i(x) = 2 = C_i$

if $\begin{cases} x_i = 2 \\ x_i' = 4 \end{cases}$

$C_i = 3,\quad f_i(x) = 0 < C_i$

if $\begin{cases} x_i = 2 \\ x_i' = 6 \end{cases}$

$C_i = 4,\quad f_i(x) = 4 = C_i$

……

if $\begin{cases} x_i = 2 \\ x_i' = 2k \end{cases}$

$C_i = k+1$, and thus $f_i(x) = 0$, if $k = 2,4,6,\ldots 2k$

$= K+1$, if $k = 1,3,5,\ldots 2k-1$

and thus $fi(x) \le C_i$

$$\text{if} \begin{cases} x_i = 4 \\ x_i' = 2k \end{cases}$$

$C_i = k+2$, and thus $f_i(x) = 0$, if k= 1,3,5,...2k -1

$= K+2$, if k = 2,4,6,...2k

$$\text{if} \begin{cases} x_i = 2\alpha \\ x_i' = 2\beta \end{cases}$$

$(\alpha, \beta, = 0,1,2,\ldots,k)$

then $C_1 = \alpha + \beta$, $f_i(x)$ 0, if $\alpha + \beta = 1,3,5,\ldots 2k-1$

$= \alpha+\beta$, if $\alpha + \beta = 0, 2, 4 \ldots 2k$

thus $f_i(x) < C_1$

Therefore , in any case, $f_i(x) \leq C1$ for every i = 1,2,3,........

Hence frome (2)

$$f\left(\frac{x_1 + x_2}{2}\right) = \sum_{i=1}^{\infty} \frac{f_i(x)}{(2k+1)^1} \leq \frac{f(x_1) + f(x_2)}{2} \qquad \text{by (1)}$$

This proves the theorem.

We now give an alternative proof of the known result of Cardinal number (which is of fundamental importance) (Kuratowski) with the help of the function f and it seems to be shorter.

Theorem 3: ($c^2 = c$) where c is the Cardinal number of the continuum.

Proof : We define the relation between the two points $x,y \in [0,1]$ by $x \rho y$ such that $x\rho y$ holds if $f(x)=f(y) = p$ where p is s fixed point of C_k (such existence has been established in our paper (Ganguly & Ray 1989).Obviously this relation is an equivalence relation. Therefore ρ partitions the interval (0,1) in to disjoint classes of equivalent elements. Lef $E_1 = \{p\}$ be the set of points of p which are left hand end points of the contiguous intervas and thus $f^{-1}(p)$ is finite for each such p (Ganguly & Ray 1989) $E_2 = \{p\}$ be the set of points p of C_k which are right hand end points of the contiguous intervals of C_k or external points and are such that $f^{-1}(p)$ is unclosed, infinite set for each such p and $E_3 = \{p\}$ be the set of points p of C_k for each of which $f^{-1}(p)$ is perfect.

$$\text{then} \left\{ \bigcup_{p \in E_1} f^{-1}(p) \right\} \cup \left\{ \bigcup_{p \in E_2} f^{-1}(p) \right\} \cup \left\{ \bigcup_{p \in E_3} f^{-1}(p) \right\} = [0,1]$$

$$\text{Hence} \sum_{p \in E_1} \overline{f^{-1}(p)} + \sum_{p \in E_2} \overline{f^{-1}(p)} + \sum_{p \in E_3} \overline{f^{-1}(p)} = [0,1] \qquad [4]$$

where $\overline{A}$ represents the Cardinal number of the set A.

$$\therefore \chi_0 + \alpha\, c + c.c = c \qquad [4]$$

where a is the Cardinal number of E_2 such that $\alpha < \chi$

$$\therefore \alpha.c + c^2 = c - \chi_0 = c$$

or $C^2 = c$.

Hence the theorem.

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Laminar free convection flow of a power law fluid from a horizontal plate.

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Abstract. Laminar free convection flow of a power law fluid from a horizontal plate has been considered. Using Karmana-Pohlhausen integral method it has been shown that similarity solutions exist if $n \neq 1$ and if the difference between the wall temperature and ambient fluid temperature varies as the cube root of the distance along the horizontal plate. The velocity and temperature distributions are obtained for different values of flow index n and parameter R_1. It has been observed that, increase in the parameter R_1 leads to decrease in the boundary layer thickness and temperature. The velocity, rate of heat transfer and shear stress at wall increases as the parameter R_1 increases in the range $0.5 \leq R_1 \leq 1.5$. The velocity at a given point within a thin liquid layer increases with the increase in the value of a vertical distance y*, but outside this layer the velocity decreases.

1. Introduction

The phenomenon of nutural or free convection arises in a fluid when changes in temperature causes density variations leading to buoyancy forces acting on the fluid elements. This process of heat transfer has many applications in heat engineering. The studies regarding laminar free convection to newtonian fluids were reported in literature by Sparrow and Minkowyez (1962) & Gill et al (1965). Kaloni (1963) has considered the free-convection flow of a visco-elastic liquid past a porous plate. The MHD free convection to visco-elastic fluids were studied by Mishra (1965), Sriramulu & Malhotra (1972) and Reddy & Bathaiah (1981). Free-convection flow of second order fluid from a horizontal plate was studied by Mishra (1968) by using the method of Karmann Pohlhausen.The same problem has bheen extended by Mishra and Roy (1970) for Waller's liquid B model. Recently Kalthia (1984) studied the laminar free convection flow of power law fluid from a vertical plate and concluded that similarity solution exists for $n \neq 1$.

The free convection flow of a Power law fluid past a horizontal plate has not been included in the above studies. It is proposed to study the free-convection effects on the flow of a power law fluid from a horizontal plate and to investigate the wall temperature, velocity distribution and boundary layer thickness which allow similarity solution.

2. Mathematical Analysis :

The boundary layer equations for the flow problem are

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = F_x - \frac{1}{\rho}\frac{\partial p}{\partial x} + \gamma\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^n \tag{1}$$

$$O = F_y - \frac{\partial p}{\rho \partial y} \tag{2}$$

$$\frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} = 0 \tag{3}$$

where F_x and F_y are body forces per unit mass acting along x- and y - axis respectively.

We assume that p does not vary across the boundary layer.

For the free convection flow from the horizontal 1 plate $F_x = 0$, $F_y = -g$

3. Solution of the problem

Since the ambient fluid is at rest, hence from equation (1), we get

$$\frac{\partial p\infty}{\partial x} = 0 \tag{4}$$

The equation of the state is

$$\rho = \rho_\infty[1 - \beta(T - T_\infty)] \tag{5}$$

wher β is coefficient of the volume expansion

From equation (2) we get

$$-\frac{\partial p}{\partial y} = \rho \infty g[1 - \beta(T - T_\infty)] \tag{6}$$

$$\frac{\partial^2 p}{\partial x \partial y} = \rho \infty g \beta \frac{\partial}{\partial x}(T - T_\infty) \tag{7}$$

Integrating the equation (7), w.r.t. y from y to delta, we obtain

$$\frac{\partial p}{\partial x} = -\rho \infty g \beta \int_y^\delta \frac{\partial}{\partial x}(T - T_\infty)\, dy \tag{8}$$

where the condition at infinity is replaced by the condition at the outer edge of the boundary.

Putting (8) in the equation, (1), leads to

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \beta g \frac{\partial}{\partial x}\int_y^\delta (T - T_\infty)\, dy + y\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^n \tag{9}$$

subject to the boundary conditions

$u = v = 0,\ T = T_w$ at $y = 0$

$$u = \frac{\partial u}{\partial y} = 0, T = T_\infty \ at\ y = \delta \tag{10}$$

Integrating equation (9) with (3) gives

$$\frac{\partial}{\partial x}\int_o^\delta u^2 dy = -\gamma[\left(\frac{\partial u}{\partial y}\right)^n]\, w + \beta\, g \frac{\partial}{\partial x}[(T_w - T_\infty) \int_o^\delta dy \int_y^\delta \frac{T - T_\infty}{T_w - T_\infty}\, dy] \tag{11}$$

Assuming

$$\theta = \frac{T - T\infty}{T_w - T_\infty},\ T_w - T_\infty = A x^m \tag{12}$$

The energy equation

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \tag{13}$$

becomes

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} + \frac{um\theta}{x} = \alpha\frac{\partial^2 \theta}{\partial y^2} \tag{14}$$

Integratin this equation w.r.t. y between 0 to delta we get

$$\frac{\partial}{\partial x}\int_0^\delta u\,\theta\, dy = -\alpha\frac{\partial \theta}{\partial y}\, w - \frac{m}{x}\int_0^\delta u\,\theta\, dy \tag{15}$$

Choose the velocity and temperature distribution for the power law fluid

$$u = u_1(x)\,\frac{y}{\delta}(1 - \frac{y}{\delta})\,\frac{n+1}{n} \tag{16}$$

and $\theta = (1 - \frac{y}{\delta})^2$ (17)

which satisfies the boundary conditions

$$u = v = 0,\ \theta = 1 \text{ at } y = 0 \tag{18}$$

$$u = 0, \frac{\partial u}{\partial y} = 0 \text{ and } \theta = 0\ at\ y = \delta \tag{19}$$

Substitution of equation (16) and (17) the equation (19) and (15) becomes

$$\frac{d}{dx}(A_1 u^2 1 \delta) = -\gamma \ \{\frac{u^1}{\delta}\}^n + \frac{1}{12} \beta \, g \, A \frac{e}{dx}(x^m \delta^2) \qquad (20)$$

and

$$B_1 (\delta \frac{du_1}{dx} + u_1 \frac{d\delta}{dx}) = \frac{2\alpha}{\delta} - \frac{B_1 m}{x} u_1 \delta \qquad (21)$$

where

$$A_1 = \frac{2n^3}{(3n+2)\,(4n+2)\,(5n+2)}$$

$$\text{and } B_1 = \frac{n^2}{(4n+1)\,(5n+1)} \qquad (22)$$

Let $u_1(x) = C_1 x^a$ and $\delta(x) = C_2 x^b$ (23)

With these, equation (20) and (21) will reduce to

$$A_1 C^2{}_1 C_2 (a+2b) x^{2a+b-1} = -\gamma \, (\frac{C_1}{C_2})^n x^{n\,(a-b)} + \frac{1}{12} \beta \, g \, A C^2{}_2 \, (2b+m) \, x^{2b+m-1} \qquad (24)$$

and

$$B_1 C_1 C_2 \, (a+b) \, x^{a+b-1} = \frac{2\alpha}{C_2} x^{-b} - B_1 m \, C_1 \, C_2 x^{a+b-1} \qquad (25)$$

Since the above equations are valid for allvalues of x.

Hence

2 a + b - 1 = n (a - b) = 2 b + m -1

and a + b - 1 = - b (26)

Solving these equations for a, b and m, we get

$$\frac{a}{1-n} = \frac{b}{1-n} = \frac{m}{1-n} = -\frac{1}{3\,(1-n)} \qquad (27)$$

since for power law fluid n ≠ 1, therefore the above relations give

$$\alpha = \frac{1}{3}, \; b = \frac{1}{3} \text{ and } m = \frac{1}{3} \qquad (28)$$

This shows that the similarity solutions exists if the difference between the wall and ambient fluid temperatures is of the form

$T_w - T_\infty = A \, x^{1/3}$ for all n

Substituting the values of a, b and m, the equations (24) and (25) reduces to

$$A_1 C_1^2 C_2 \;=\; -\gamma\left(\frac{C_1}{C_2}\right)^n + \frac{1}{12}\beta\, g\, A\, C_2^{\,2} \qquad (29)$$

and

$$B_1\, C_1\, C_2 \;=\; 2\,\frac{\alpha}{C_2} \qquad (30)$$

For any given situation, only the unknowns are C_1 And C_2, which can be determined from the equations (28) and (29). Eliminating C_1 from the equations (28) and (29), we get

$$\lambda^5 - \left(\frac{2}{B_1}\right)^n P_r\, R_1^{\frac{3n-3}{5}}\, \lambda^{3-3} - \frac{4A_1}{B_1^2} \;=\; 0 \qquad (31)$$

where

$$\lambda \;=\; \left(\frac{\beta\, g\, A.}{12\alpha^2}\right)^{1/5} C_2$$

$$R_1 \;=\; \frac{\beta\, g\, A}{12\alpha^2}$$

and $P_r \;=\; \dfrac{\gamma}{\alpha^{2-n'}}$, modified Prandt' 1 number (32)

λ is real root of the equation (30), which has only one positive root.

For given values of the parmeter R_1 and Prandt' 1 number P_r, the value of λ are determined from the equation (31) for different values of the flow index n.

The value of C_1 is determined from the equation (30) as

$$\frac{C_1}{\alpha}\left(\frac{12\alpha^2}{\beta\, g\, A}\right)^{2/5} \;=\; C_1^{*} = \frac{1}{\lambda^2} \qquad (33)$$

the velocity and temperature distribution can be obtained from the following equations

$$\frac{u^*}{x^*} \;=\; \frac{y^*}{\lambda^3}\left(1 - \frac{y^*}{\lambda}\right)^{\frac{n+1}{n}} \qquad (34)$$

$$\text{and } \theta \;=\; \left(1 - \frac{y^*}{\lambda}\right)^2 \qquad (35)$$

where

$$x^* = x^{2/3}\left(\frac{\beta g A}{12\alpha^2}\right)^{1/5}, \quad y^* = \frac{y}{x^{1/3}}\left(\frac{\beta g A}{12\alpha^2}\right)^{1/5}$$

$$\text{and } u^* = \left(\frac{B_1}{2\alpha}\right)\left(\frac{u}{x^{1/3}}\right)\left(\frac{12\alpha^2}{\beta g A}\right)^{1/5} \tag{36}$$

4. Discussion

The values of λ and C_1^* obtained for $P_r = 10$ different values of parameter R_1 and flow index n are presented in table 1. It is evident that the value of λ is decreasing in the range $0.5 \leq R_1 \leq 1.5$ when the value of the parameter R_1 is increasing. This shows that the increase in R_1 leads to decrease in the boundary layer thickness.

The shearing stress of the wall is given by

$$\tau_W = K\left(\frac{\partial u}{\partial y}\right)\frac{n}{y} = 0$$

$$= K\left(\frac{C_1}{C_2}\right)^n$$

$$= K\left[\frac{2\alpha}{B_1 \lambda^3}\left(\frac{\beta g A}{12\alpha^2}\right)^{3/5}\right]^n \tag{37}$$

$$\text{Let } \tau_W^* = \left(\frac{.\dot{2}}{B_1 . \lambda^3}\right)^n$$

The values of τ_W^* are given in table 1. It is found that increase in R_1 leads to increase in the shear stress for the range $0.5 \leq R_1 \leq 1.5$. The shear stress at $R_1 = 0$ is greatest.

The Nusselt number N_{u_x} is defined as

$$N_{u_x} = \frac{-\left(\frac{\delta T}{\delta y}\right)_{y=0} . x}{(T_W - T_\infty)} \tag{38}$$

$$N_{u_x} = \frac{2x^{2/3}}{C_2}$$

$$\text{i.e. } N_{u_x} = \frac{2x^*}{\lambda}$$

The table 1 shows the values of the Nusselt number N_{u_x} for given value of $P_r = 10$

and different values of parameter R_1 and flow index n. It is to be noted that increase in R_1 yields increase in the Nusselt number for the range $0.5 \leq R_1 \leq 1.5$. The Nusselt number is greater when R_1 is zero.

The velocity distribution obtained for $P_r = 10$ and the different values of the parameter R_1 and flow index n have been presented in tables 2-5. It is found that the velocity at a given point within a thin liquid layer increases with the increase in the value of y*, but outside this layer the velocity decreases. The minimum value of u* occurs at y* = 0 and y* = λ, i.e. at the plate and at the edge of the boundary layer. The velocity is maximum near y* = λ/3. The velocity is maximum when $R_1 = 0$. When R_1 increases in the range $0.5 \leq R_1 \leq 1.5$, The velocity increases for all $n \leq 1.5$ but at n = 1.8 the velocity decreases.

For fixed value of parameter R_1, The velocity increases as the flow index n increases but in the beginning just opposite nature has been observed.

The temperature distribution has been given in tables 6-9 for different values of parameter R_1 and flow index n, with $P_r = 10$. It is evident that the temperature decreases when R_1 increases in the range $0.5 \leq R_1 \leq 1.5$. Also when the distance y* increases the tempetature decreases. For $R_1 = 0$, the temperature increases when n increases, but for the range $0.5 \leq R_1 \leq 1.5$,the temperature decreases as n increases.

5. Conclusions :

(1) Increase in the parameter R_1 leads to decrease in the boundary layer thickness.

(2) The velocity increases as R_1 increases.

(3) The increase in the value of R_1 leads to fall in the temperature.

(4) As n increases the temperature decreases for the range $0.5 \leq R_1 \leq 1.5$.

(5) Increase in R_1, increases the nusselt number for the range $0.5 \leq R_1 \leq 1.5$.

(6) Increase in R_1 leads to increase in the shear stress at the wall for the range $0.5 \leq R_1 \leq 1.5$.The shear stress has maximum value when $R_1 = 0$.

Table 1: Values of shear stress and Nusselt number for $P_r=10$

R1	n	λ	C_1^*	τ_w^*	Nu_x
0	0.5	1.9473	22.1520	3.2728	1.0271
	0.8	2.0015	16.3817	5.3753	0.9992
	1.5	2.0723	12.3157	14.4880	0.9651
	1.8	2.0904	11.5836	21.8022	0.9568
0.5	0.5	4.2024	4.7565	1.0639	0.4759
	0.8	3.9498	4.2064	1.0516	0.5063
	1.5	3.6336	4.0058	1.1574	0.5504
	1.8	3.5540	4.0073	1.2419	0.5627
1.0	0.5	3.6802	6.2022	1.2982	0.5435
	0.8	3.6529	4.9178	1.2685	0.5475
	1.5	3.5969	4.0879	1.2116	0.5560
	1.8	3.5795	3.9505	1.1942	0.5587
1.5	0.5	3.4129	7.2113	1.4536	0.5860
	0.8	3.4925	5.3802	1.4129	0.5727
	1.5	3.5756	4.1367	1.2444	0.5593
	1.8	3.5945	3.9176	1.1676	0.5564

Table 2: Velocity distribution for a =0.5, Pr=10

Y^*/R_1	0	0.5	1.0	1.5
0.0	0	0	0	0
0.1	0.0116	0.0012	0.0018	0.0023
0.2	0.0196	0.0023	0.0034	0.0042
0.3	0.0246	0.0032	0.0047	0.0057
0.4	0.0272	0.0040	0.0057	0.0069
0.5	0.0278	0.0046	0.0065	0.0078
0.6	0.0269	0.0051	0.0071	0.0084
0.7	0.0249	0.0055	0.0075	0.0088
0.8	0.0222	0.0057	0.0077	0.00903
0.9	0.0189	0.0059	0.0078	0.00904
1.0	0.0155	0.0060	0.0077	0.0089
1.1	0.0123	0.0061	0.0076	0.0086
1.2	0.0918	0.0059	0.0074	0.0082
1.3	0.0064	0.0058	0.0071	0.0078
1.4	0.0042	0.0056	0.0067	0.0072
1.5	0.0025	0.0054	0.0063	0.0066

Table 3: Velocity distribution for n=0.8, Pr=10

y^*/R_1	0	0.5	1.0	1.5
0.0	0	0	0	0
0.1	0.0111	0.0015	0.0019	0.0022
0.2	0.0197	0.0029	0.0036	0.0041
0.3	0.0259	0.0041	0.0051	0.0057
0.4	0.0302	0.0051	0.0063	0.0071
0.5	0.0327	0.0060	0.0074	0.0083
0.6	0.0336	0.0067	0.0082	0.0092
0.7	0.0331	0.0073	0.0089	0.0099
0.8	0.0316	0.0078	0.0094	0.0104
0.9	0.0293	0.0082	0.0098	0.0108
1.0	0.0263	0.0084	0.00999	0.0109
1.1	0.0228	0.0085	0.0101	0.0110
1.2	0.0191	0.0086	0.0100	0.0109
1.3	0.0153	0.00859	0.0099	0.0107
1.4	0.0117	0.0085	0.0097	0.0104
1.5	0.0083	0.0083	0.0094	0.0099

Table 4: Velocity distribution for n=1.5,Pr=10

y^*/R_1	0	0.5	1.0	1.5
0.0	0	0	0	0
0.1	0.0103	0.0020	0.00205	0.0021
0.2	0.0190	0.0038	0.0039	0.0040
0.3	0.0260	0.0054	0.0056	0.0057
0.4	0.0314	0.0069	0.0071	0.0072
0.5	0.0355	0.0081	0.0084	0.0085
0.6	0.0381	0.0092	0.0095	0.0097
0.7	0.0396	0.0102	0.0105	0.0106
0.8	0.0399	0.0110	0.0113	0.0115
0.9	0.0391	0.0117	0.0120	0.0121
1.0	0.0375	0.0122	0.0125	0.0127
1.1	0.0350	0.0126	0.0129	0.0130
1.2	0.0319	0.0128	0.0131	0.0133
1.3	0.0129	0.01323	0.01323	0.0134
1.4	0.0241	0.0130	0.0132	0.01338
1.5	0.0197	0.0129	0.0131	0.0132

Table 5: Velocity distribution for u=1.5, Pr=10

y^*/R_1	0	0.5	1.0	1.5
0.0	0	0	0	0
0.1	0.0101	0.0021	0.00209	0.00206
0.2	0.0187	0.0041	0.0040	0.0039
0.3	0.0258	0.0058	0.0057	0.0056
0.4	0.0315	0.0074	0.0072	0.0072
0.5	0.0358	0.0088	0.0086	0.0085
0.6	0.0389	0.0100	0.0098	0.0097
0.7	0.0406	0.0111	0.0109	0.0107
0.8	0.0413	0.0120	0.0118	0.0116
0.9	0.0410	0.0127	0.0125	0.0124
1.0	0.0398	0.0133	0.0131	0.0130
1.1	0.0377	0.0138	0.0135	0.0134
1.2	0.0348	0.0141	0.0139	0.0137
1.3	0.0313	0.0142	0.0140	0.0139
1.4	0.0273	0.0143	0.0141	0.0140
1.5	0.0230	0.0142	0.0140	0.0139

Table 6: Temperature distribution forn = 0.5,Pr = 10

y^*/R_1	0	0.5	1.0	1.5
0.0	1	1	1	1
0.1	0.8999	0.9529	0.9464	0.9423
0.2	0.8051	0.9071	0.8943	0.8862
0.3	0.7156	0.8623	0.8436	0.8319
0.4	0.6314	0.8187	0.7944	0.7793
0.5	0.5523	0.7762	0.7467	0.7285
0.6	0.4787	0.7348	0.7005	0.6793
0.7	0.4103	0.6946	0.6558	0.6319
0.8	0.3471	0.6555	0.6125	0.5861
0.9	0.2892	0.6175	0.5707	0.5421
1.0	0.2366	0.5807	0.5304	0.4998
1.1	0.1893	0.5450	0.4915	0.4593
1.2	0.1473	0.5104	0.4542	0.4204
1.3	0.1105	0.4770	0.4183	0.3833
1.4	0.0790	0.4447	0.3839	0.3479
1.5	0.0528	0.4135	0.3509	0.3142

Table 7: Temperature distribution for n=0.8,Pr=10

y^*/R_1	0	0.5	1.0	1.5
0.0	1	1	1	1
0.1	0.9026	0.9500	0.9460	0.9435
0.2	0.8101	0.9013	0.8935	0.8887
0.3	0.7227	0.8539	0.8425	0.8356
0.4	0.6402	0.8077	0.7930	0.7841
0.5	0.5628	0.7628	0.7450	0.7342
0.6	0.4903	0.7133	0.6985	0.6859
0.7	0.4228	0.6770	0.6535	0.6393
0.8	0.3603	0.6359	0.6099	0.5943
0.9	0.3029	0.5962	0.5679	0.5510
1.0	0.2504	0.5577	0.5274	0.5093
1.1	0.2029	0.5206	0.4884	0.4693
1.2	0.1603	0.4847	0.4509	0.4309
1.3	0.1228	0.4501	0.4149	0.3941
1.4	0.0903	0.4167	0.3104	0.3590
1.5	0.0628	0.3847	0.3474	0.3255

Table 8: Temperature distribution for n= 1.5,Pr=10

y^*/R_1	0	0.5	1.0	1.5
0.0	1	1	1	1
0.1	0.9058	0.9457	0.9452	0.9448
0.2	0.8163	0.8929	0.8919	0.8913
0.3	0.7314	0.8417	0.8401	0.8392
0.4	0.6512	0.7919	0.7899	0.7888
0.5	0.5756	0.7437	0.7413	0.7399
0.6	0.5048	0.6970	0.6942	0.6925
0.7	0.4385	0.6518	0.6486	0.6468
0.8	0.3769	0.6081	0.6046	0.6026
0.9	0.3200	0.5660	0.5622	0.5599
1.0	0.2677	0.5253	0.5212	0.5189
1.1	0.2201	0.4862	0.4419	0.4794
1.2	0.1772	0.4486	0.4441	0.4414
1.3	0.1389	0.4124	0.4078	0.4050
1.4	0.1052	0.3779	0.3730	0.3702
1.5	0.0763	0.3448	0.3398	0.3370

Table 9: Temperature distribution for n=1.8,Pr=10

y^*/R_1	0	0.5	1.0	1.5
0.0	1	1	1	1
0.1	0.9066	0.9445	0.9449	0.9451
0.2	0.8178	0.8906	0.8914	0.8918
0.3	0.7336	0.8383	0.8394	0.8400
0.4	0.6539	0.7876	0.7890	0.7898
0.5	0.5788	0.7384	0.7401	0.7411
0.6	0.5083	0.6908	0.6928	0.6940
0.7	0.4424	0.6449	0.6471	0.6484
0.8	0.3810	0.6005	0.6030	0.6044
0.9	0.3243	0.5577	0.5603	0.5619
1.0	0.2721	0.5164	0.5193	0.5210
1.1	0.2245	0.4768	0.4798	0.4816
1.2	0.1814	0.4387	0.4419	0.4438
1.3	0.1430	0.4022	0.4055	0.4075
1.4	0.1091	0.3673	0.3707	0.3727
1.5	0.0798	0.3340	0.3375	0.3395

Nomenclature

x: coordinate along the horizontal plate
y: coordinate along the normal to the plate
u: horizontal component of the velocity
v: vertical component of the velocity
g: acceleretion due to gravity
Fx Fy: body forces along x-and y-directions
P: Pressure
C_p: specific heat at constant pressure
K: power law fluld parameter
n: power law fluid index
T: Temperature of the fluid
Tw: Temperature of the wall
T_∞: Temperature at infinity
A_1, B_1: constants defined in equation (21)
R_1: parameter defInded in equation (31)

A,Cl, C2 C*1: Constants

Pr: modified prandt'1 number

N_{u_x}: Nusselt number

k: thermal conductivity

Greek symbol

δ : boundary layer thickness

ρ : density

ρ_∞ : density of the ambient fluid

β : coefficient of the volume expansion

α : thermal diffusivity, $\frac{k}{\rho C_p}$

γ : coefficient of the viscosity

θ : dimensionless temperature, $\frac{T - T_\infty}{T_w - T_\infty}$

λ : constant parameter defined in equation (31)

τ_w : shear stress at the wall,

τ_w^* : proportional shear stress

Subscripts

∞ : condition far away from the plate

w : condition at the wall

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Necessary optimality criteria on a Banach space

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Abstract : Let X be a real Banach space and let X^* be its dual space. Let $f : X \rightarrow R$ and $g : X \rightarrow R^m$ be Frechtt differentiable functions defined on an open set $X^0 C X$. Let the maximizing problem be as follows:

To find $X \in X'$ *such that* $f(X) \leq f(\bar{x})$, $\forall x \in X'$

where $X \in X' = \{X : X \in X^0 \cdot g(x) \geq 0\}$.

The object of this paper is to deduce first Fritz - John type of necessary conditions for the above maximizing problem with the help of strict separation theorem for closed convex cones in Banach space and them Kuhn- Tucker types of necessary conditions under suitable constraint qualifications.

1. Introduction

Necessary conditions for the above type of maximizing problem have been investigated by many mathematicians like variya (1967) Ritter, (1969), Bhakta, and Roychaudhury (1988), Varaiya (1967), has considered $f \cdot X \rightarrow R$, $g : X \rightarrow Y$, X and Y being real Banach spaces; Ritter, K; Bhakta and Roychaudhury have considered $f : X \rightarrow Y$, $g : X \rightarrow Y^m$, $Y^m = Y \times Y \times Y_x \ldots\ldots \times Y$. Bhakta and Roychaudhury (1988) have obtained Fritz - John type of necessary conditions by using a strict separation theorem in Banach space as an axiom; They have used the concept of polar to obtain necessary conditions for Kuhn - Tucker type of problems. Varaiya, has used the concept of attainable directions, locally constrained directions, weak and kuhn - Tucker constrained qualifications (the latter being different from that used by Mangasarian 1969) to obtain Kuhn - Tucker type of necessary conditions.

We have, in this paper, used strict separation theorem for convex cones (Dunford & Schwart 1958) and the method of Bhakta and Roychaudhury (1988) for deducing Fritz - John type of necessary conditions. Lastly, using some constraint qualifications and proceeding as in Mangasarian, we have obtained Kuhn - Tucker type of necessary conditions.

2. Some notations and definitions

Let $a_1^* \in X^*$, $i = 1, 1, 2 \ldots\ldots\ldots m$; X^* being the dual of a real Banach space X. We call the matrix

$$A = \begin{bmatrix} a_1^* \\ a_2^* \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ a_m^* \end{bmatrix}$$

as a m x X* matrix.
for x ∈ X, we write

$$Ax = \begin{bmatrix} a_1^* x \\ a_2^* x \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ a_m^* x \end{bmatrix} \in R^m$$

For $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_m) \in R^m$,

we write $\lambda A = \sum_{i=1}^{m} \lambda_i \ a_i^*$.

2.1. Definition

It is known from (Ritter 1969) that a vector $z \in X$ is an attainable direction at $\bar{x} \in X$ if there exists an arc $e\ \theta$ $(0 \le \theta \le 1)$such that

(i) $e(0) = \bar{X}$

(ii) $e(\theta)$ is differentiable from right at $\theta = 0$

(iii) $\frac{de}{d\theta} \Big/ \theta = 0 + = z$

3. Theorems on alternatives

3.1. Farkas Lemma Form I

Let A be a m x X matrix defined by

$$A = \begin{bmatrix} a_1^* \\ a_2^* \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ a_m^* \end{bmatrix}, a_i^* \in X^*$$

if b $\in X^*$,

$$\left\langle \begin{matrix} A_x \le 0 \\ \forall_x \in X \end{matrix} \Rightarrow b^* x \le 0 \right\rangle \Leftrightarrow \left\langle \exists\ \lambda \in R_m,\ \lambda \ge 0 \text{ such that } b^* = \lambda A \right\rangle$$

Proof : To show L.H.S. = R.H.S.

If $b^* = (0, 0, \ldots, 0)$, then obviously R.H.S. holds, by taking $\lambda = 0$, whatever maybe A. So, let $b^* \neq (0, 0, \ldots, 0)$ and that there does not exist any $\lambda \ge 0$ such that $b^* = \lambda A$.

Now let $C_1 = \{ a^*, a^* \in X^* / a = \lambda A, \lambda \in R^m, \lambda \ge 0\}$

and $C_2 = \{b^*\}$

Since X^* is a linear space over R, so C_1 is nonempty. By hypothesis C_2 is nonempty. C_1 and C_2 are disjoint as

$b^* \neq (0, 0 \ldots, 0)$.

C_1 is a cone for $\alpha > 0$ $(\alpha \in R)$, $a^* \in C_1 \Rightarrow \alpha a^* \in C_1 \ \forall a^* \in C_1$

in fact

$$\alpha a^* = \alpha(\lambda A) = \alpha(\lambda_1 a_1{}^* + \lambda_2 a_2{}^* + \ldots\ldots\ldots\ldots \lambda_m a_m^*)$$

$$= \Sigma\ (\alpha \lambda_i)\, a^*{}_i = \Sigma\, \mu_i a^*{}_i = \mu A,$$

say, where $\mu \ge 0$.

Obviously C_1 is convex. Lastly, we show that C_1 is closed. We consider a mapping p : $C_1 \rightarrow R^m$ s.t. $p(\lambda A) = \lambda$

Now considering usual topology on C_1 and usual product topology on R^m, p is found to be continuous.

Further the set $\wedge$ $(\subset R^m) = \{\lambda/\lambda \in R^m, \lambda \ge 0\}$ is a closed set. Hence C_1 is closed.

Now by applying strict separation theorem (Dunford and Schwartz 1958) on the disjoint subsets C_1 and C_2 of X^* there exists an element $z\ (\neq 0) \in X$ such that

$b^* z > 0$ .... (3.1.1)

$a^* z \le 0 , \forall a^* \in C_1$ .... (3.1.2)

So, $\lambda Az \le 0 , \lambda \ge 0$ since $a^* = \lambda A$

Let us choose $\lambda = (O, 0, \ldots, \lambda_i, \ldots\ldots 0)$, $\lambda_i > 0$

Then $a_{i^*} z \le 0$.

Repeating this choice for i = 1, 2, ..., m,

we have $Az \le 0$. ... ... (3.1.3)

(3.1.1) and (3.1.3) taken together gives rise to a contradiction as in such a case

$\langle Ax \le 0 \Rightarrow b^* x \le 0 \rangle$ can not hold.

$\forall \alpha \in x$

So, L.H.S. ⇒ R.H.S.

It is immediate that R.H.S.⇒ L.H.S.

3.2. Farkas Lemma Form II

$$\text{Let} \quad A = \begin{bmatrix} a_1^* \\ a_2^* \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ a_m^* \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} b_1^* \\ b_2^* \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ b_n^* \end{bmatrix}$$

be respectively m x X* and n x X* matrix.

if $b^* \in X^*$, then

$$\left\langle \begin{array}{l} Ax \le 0, Bx = 0 \Rightarrow b^* X \le 0 \\ \forall x \in x \end{array} \right\rangle \Leftrightarrow \left\langle \begin{array}{l} \exists (\lambda, \mu) \in R^m \times R^n \quad \lambda \le 0 \\ \text{s.t.} \quad b^* = \lambda a + \mu B \end{array} \right\rangle$$

Proof : To show L.H.S. ⇒ R.H.S.

If $b^* = (0, 0, \ldots, 0)$, then the result holds where $\lambda \ge 0$, $\mu \ge 0$.

So, let $b^* \ne (0, 0, \ldots, 0$ and if possible, let there be no

$(\lambda , \mu) \in R^m \times R^n , \lambda \ge 0$ *s.t.* $b^* = \lambda A + \mu B$.

Let $C_1 = \{ a^* a^* \in X^* / a^* = \lambda A + \mu B, \ (\lambda , \mu) \in R^m \times R^n \ , \lambda \ge 0 \}$

and $C_2 = \{ b^* \}$

As in form I, if can be shown that C_1 is a closed convex cone.

So, by strict separation theorem, there exists an element

$z \ (\neq 0) \in X$ such that

$b^* z > 0,$ .... (3.2.1)

$a^* z \le 0, \quad \forall \ a^* \in C_1$ .... (3.2.2)

So, $\lambda Az + \mu Bz \le 0, \ (\lambda, \mu) \in R^m \times R^n, \lambda \ge 0$ .... (3.2.3)

Putting $\mu = 0, \quad \lambda A z \le 0$

Choosing $\lambda = (0, 0, \ldots, \lambda_i, 0, \ldots, 0), \qquad \lambda_i > 0$

$a_i^* z \le 0$

Repeating this choice for i = 1, 2, . . . , m, we have

$Az \le 0$ .... (3.2.4)

Again putting $\lambda = 0$ in (3.2.3), we have, $\mu Bz \le 0$.

So $Bz = 0$ as μ is arbitrary. .... (3.2.5)

Now (3.2.1), (3.2.4), (3.2.5) taken together leads to a contradiction. Hence L.H.S.$\Rightarrow$ R.H.S.

The converse part is immediate.

This completes the proof.

3.3. Theorem on alternative form I

Let A be a m x X* matrix. Then either there exists a vector x in X such that $Ax < 0$ or there exists a vector $\lambda \in R^m \ (\lambda \ge 0)$ such that $\lambda A = 0$, but never both.

Theorem on alternative form II

Let A be a m x X* matrix and B be a n x X* matrix. Then either there exists a vector x $\in$ X such that $A x < 0$, $B_x = 0$, $B_x = 0$ or there exists a vector $(\lambda, \mu) \in R^m \times R^n, \lambda \ge 0$ such that

$\lambda A + \mu B = 0$, but never both.

Proof : Let $C_1 = \{a^* / a^* = \lambda A + \mu B, \ (\lambda, \mu) \in R^m \times R^n, \ \lambda \ge 0\}$ since λA and μB belong to X* , so for $\lambda \ge 0$ and A, B being given, it is possible to choose some μ (μ being arbitrary) such that $\lambda A + \mu B = 0 \in X^*, \ (\lambda, \mu) \in R^m \times R^n, \ \lambda \ge 0$.

Otherwise $a^* \neq 0, \quad \forall a^* \in C_1$.

Now C_1 is a closed convex cone of X* , not containing $0 \in X^*$.

So, there exists $z\ (\neq 0) \in X$ such that $a^* z < 0, \ \forall a^* \in C_1$.

So, $\lambda Az + \mu Bz < 0, \ \lambda \geq 0$

putting $\mu = 0$, we get $\lambda Az < 0, \ \lambda \geq 0$.

We first choose $\lambda = (0, 0, \ldots, \lambda_i, 0, \ldots, 0), \quad \lambda_i > 0$.

Then $a_i{}^* z < 0$.

Repeating this choice for $i = 1, 2, \ldots, m$, we get

$a_{i^*} z < 0, \qquad i = 1, 2, \ldots, m$.

i.e. $Az < 0$.

Next we choose $\lambda \geq 0$, any $\mu \in R^n$ and $s > 1$, any positive integer.

Then we have $\lambda Az + s \ \mu Bz < 0$, as μ is arbitrary.

i.e. $\mu B(z) < -\dfrac{\lambda Az}{s}$.

Taking $s \to \infty$, we get $\mu B(z) \leq 0$.

As μ is arbitrary, so $B(z) = 0$.

Thus we get some $z\ (\neq 0) \in X$ such that $Az < 0, \ Bz = 0$.

This completes the proof.

4. Fritz - John necessary optimality criteria

Theorem 4.1

Let X^0 be an open set of a real Banach space X.

Let $f : X^0 \to R$ and $g = (g_1, g_2, \ldots, g_m) : X^0 \to R^m$ be Frecht differentiable. Let $\bar{x} \in X^0$ solve the following Local maximizing problem (IMP).

$f(x) \leq f(\bar{x}), \ \forall x \in x' \bigcap B_\partial(\bar{x})$

where $\bar{x} \in x' = \{x : x \in X^0 \text{ s.t. } g(x) \geq 0\}$

and $B_\delta(\bar{x})$ is an open ball of radius $\bar{\delta} > 0$ with centre $\bar{x}$;

then $\exists$ a vector $(\bar{\lambda}_o, \bar{\lambda}) \in R \times R^m$ such that

$$\overline{\lambda_o} \nabla f(\bar{x}) + \overline{\lambda_o} \nabla g(\bar{x}) = 0, \quad (\overline{\lambda_o}, \bar{\lambda}) \geq 0.$$

Proof : Let $I = \{i : g_i(\bar{x}) = 0\}$

$J = \{i : g_i(\bar{x}) > 0\}$

Let $I = \{1, 2, 3, \ldots, r\}$ and $J = \{r+1, r+2, \ldots, m\}$

We first show that

$$\left\langle \begin{array}{c} \nabla f(\bar{x})z > 0 \\ \nabla g_I(\bar{x})z > 0 \end{array} \right\rangle \qquad \ldots\ldots \qquad \ldots\ldots \qquad (4.1)$$

has no solution $z \in X^0$, where $g_I = (g_1, g_2, \ldots g_r)$.

If possible, let (4.1) have a solution for some $z \in X^0$.

Now X^0 being open, $\exists\ \delta > 0$ such that for $0 < \delta < \hat{\delta}$

$\bar{x} + \delta z \in X^0$. since f and g are Frecht differentiable

at $\bar{x}$, we have, for $0 < \delta < \overset{\Delta}{\delta}$,

$f(\bar{x} + \delta z) = f(\bar{x}) + \delta \nabla f(\bar{x})z + \alpha_0(\bar{x}, \delta z)\delta \,||\, z \,||$

$g_i(\bar{x}) + \delta z) = g_i(\bar{x}) + \delta \nabla g_i(\bar{x})\, z + \alpha_i(\bar{x}, \delta z)\delta \,||\, z \,||,\ i = 1,2,\ldots,m$

Where $\lim\limits_{\delta \to 0} \alpha_i(\bar{x}, \delta z) = 0,\ i = 0,1,2,\ldots,m.$

Now choosing δ sufficiently small, $0 < \delta < \delta_0$, say

$f(\bar{x} + \delta z) - f(\bar{x}) > 0$ [*since* $\nabla f(\bar{x})\, z > 0$].

Next choosing δ sufficiently small,$0 < \delta\ \delta_i,\ i \in I$

$g_i(\bar{x} + \delta z) - g_i(\bar{x}) > 0$ [*since* $\nabla g_I(\bar{x}) > 0$]

Lastly choosing δ sufficiently small, $0 < \delta < \delta_i,\ i \in J,$

$g_i(\bar{x} + \delta z) > 0$ [*since* $g_i\bar{x} > 0\ \ i \in j$].

Hence $g(\bar{x} + \delta z) > 0$, δ being sufficiently small.

Thus $\bar{x} + \delta z \in X'$.*Also* $\bar{x} + \delta z \in B_\delta(\bar{x})$

But $f(\bar{x} + \delta z) > f(\bar{x})$, *so* $\bar{x}$ can not solve LMP. Hence we get a contradiction.

Thetefore,if we consider

$$A = \begin{bmatrix} -\nabla f(\bar{x}) \\ -\nabla g_1(\bar{x}) \\ -\nabla g_2(\bar{x}) \\ . \\ . \\ . \\ -\nabla g_n(\bar{x}) \end{bmatrix}$$

We see that $Az < 0$ has no solution $z \in X^{0.}$

Hence by theorem on alternative, form I, $\exists (\bar{\lambda}, \bar{\lambda}_I) \in R \times R^{I}$

such that $\bar{\lambda}_0 \nabla f(\bar{x}) + \bar{\lambda}_I \nabla g_I(\bar{x}) = 0$, $(\bar{\lambda}_0. \bar{\lambda}_I) \geq 0$

choosing $\bar{\lambda}_i = 0$, $i \in J$, we have,

$\bar{\lambda}_0 \nabla f(\bar{x}) + \bar{\lambda}_I \nabla g_I(\bar{x}) + \bar{\lambda}_J \nabla g_J(\bar{x}) = 0$,

$(\bar{\lambda}_0, \bar{\lambda}_I), \geq 0$, $\bar{\lambda}_J = 0$.

Thus $\bar{\lambda}_0 \nabla f(\bar{x}) + \bar{\lambda} \nabla g(\bar{x}\}) = 0$, $(\bar{\lambda}_0, \bar{\lambda}) \in R x R^m$, $(\bar{\lambda}_0, \bar{\lambda}_I) \geq 0$

Further $\bar{\lambda} g(\bar{x}) = \bar{\lambda}_I g_I(\bar{x}) + \bar{\lambda}_J g_J(\bar{x}) = 0$

This completes the proof.

5. Kuhn - Tucker type of necessary conditions

Theorem 5.1

Let $X^{0.}$ f, g be defined as in Theorem 4.1; let $(\bar{x})$solve the following local maximizing problem :

$f(x) \leq f(\bar{x})$, $\forall x \in x' \cap B\delta(\bar{x})$, where

$\bar{x} \in X' =, \{x : x \in X^0, g(x) > 0\}$,

Then there exists $\bar{\lambda} \in R^m$, $\bar{\lambda} \geq$ s.t. $\nabla f(\bar{x}) + \bar{\lambda} \nabla g(\bar{x}) = 0$.

Proof : Choose any $Y \in X^0$; then X^0 being open, $\bar{x} + \delta Y \in X^0$, $\delta > 0$

Now $g_i(\bar{x} + \delta Y) = g_i(\bar{x}) + \delta [\nabla g_i(\bar{x}) Y + \alpha_i(\bar{x}, \delta Y)]$,

$i = 1, 2, \ldots, m$

Also $f(\bar{x} + \delta Y) = f(\bar{x}) + \delta [\nabla f(\bar{x}) Y + \alpha_0(\bar{x}, \delta Y)]$

Hence for sufficiently small δ, $g_i(\bar{x} + \delta Y) > 0$.

Hence $\bar{x} + \delta Y \in x'$; also $\bar{x} + \delta y \in B_{\delta}(\bar{x}))$

As $\bar{x}$ solves the above local maximizing problem, so

$f(\bar{x} + \delta Y) \leq f(\bar{x})$, for small choice of δ.

In such cases, $\nabla f_i(\bar{x}) Y \leq 0$.

Now if we start with $\bar{x} - \delta Y \in X^0$, $\delta < 0$, then arguing

as above, we have $\nabla f(\bar{x}) Y \geq 0$. *Thus* $\nabla f(\bar{x}) Y = 0$.

But $\nabla f(\bar{x}) : x \rightarrow x*$ (dual space of X) is a linear map

and Y is arbitary, so $\nabla f(\bar{x}) Y = 0 \Rightarrow \nabla f(\bar{x})\ 0$.

So choosing $\bar{\lambda} = 0$, *we get* $\nabla f(\bar{x}) + \bar{\lambda}\, \nabla g(\bar{x}) = 0$.
This completes the proof.

Theorem 5.2

Let X^0, f, g be defined as in Theorem 4.1 and let $\bar{x}$ solve the local maximizing problem as defined there; further let there exist some $z \in X^0$ such that $\nabla g_I(\bar{x})\; z < 0$, where $I = \{ i : g_I(\bar{x}) = 0\}$. Then there exists $\bar{\lambda} \in R^{m\cdot}\; \bar{\lambda} \geq 0$ such that

$\nabla f(\bar{x}) + \bar{\lambda}\, \nabla g(\bar{x}) = 0$

Proof : As in Theorem 4.1, it may be shown that there exist
$(\bar{\lambda}_o, \bar{\lambda}_I, \bar{\lambda}_J) \in R \times R^r \times R^{m-r}$, $(\bar{\lambda}_o, \bar{\lambda}_I) \geq 0$, $\bar{\lambda}_J = 0$ s.t.
$\bar{\lambda}_o \nabla f(\bar{x}) + \bar{\lambda}_I \nabla g_I(\bar{x}) + \bar{\lambda}_J \nabla g_J(\bar{x}) = o.$
We now show that $\bar{\lambda}_o \neq 0$; if possible let $\bar{\lambda}_0 = 0$, then $\bar{\lambda}_I \neq 0$,
and we get $\lambda_I \nabla g_I(\bar{x}) = 0$ [*since* $\bar{\lambda}_J = 0$]
So, $\bar{\lambda}_I \nabla g_I(\bar{x})\, z = 0$, contrary to the hypothesis that $\nabla g_I(\bar{x})\, z < 0$.
This proves the Theorem.

Theorem 5.3

Let X^0, f, g be defined as in Theorem 4.1 and $\bar{x}$ solve the local maximizing problem defined there; further let

$$\left\langle \begin{matrix} g_I(\bar{x})\, z \geq 0 \\ z \in X^0 \end{matrix} \right\rangle \Rightarrow z \text{ is an attainable direction at } \bar{x} \in X^0,$$

where $I = \{ i : g_I(\bar{x}) = 0\}$. Then there exists $\bar{\lambda} \in R^m$, $\bar{\lambda} \geq 0$

such that $\nabla f(\bar{x}) + \bar{\lambda}\, \nabla g(\bar{x}) = 0$.

Proof : We have,

$e(\theta) = e(0) + \theta\, e'(0) + 0(\theta) = \bar{x} + \theta z + 0(\theta)$.

Let N be an arbitrary neighourhood of $\bar{x}$ and θ (N) be sufficiently small so that $e(\theta) \in X^0 \cap N$, $\forall\ \theta \leq (N)$

since $\bar{x}$ solves the local maximizing problem, therefore

$$0 \geq f[\, e(\theta)\,] - f[\, e(0)\,] = [\, \nabla f(\, e(0) \frac{de(0)}{d\theta}\,]\, \theta + \beta(0, \theta)$$

where $\lim_{\theta \to 0} \beta(0, \beta) = 0$.

Hence taking θ sufficiently small,

$$\nabla f[\, e(\theta)\,] \frac{de(0)}{d\theta} \leq 0.$$

i.e. $\nabla f(\bar{x})\, z \leq 0$.

Let $I_1 = \{1, 2, \ldots, t\}$ such that $\nabla g\, I_1(\bar{x})\, z > 0$

$I_2 = \{t+1, t+2, \ldots, r\}$ *such that* $\nabla g\, I_2(\bar{x})\, z = 0$

and $I = I_1 \bigcup I_2$.

Thus $\nabla g_I(\bar{x})\, z \geq 0 \Rightarrow \nabla f(\bar{x})\, z \leq 0$.

writing

$$A = \begin{bmatrix} -\nabla f(\bar{x}) \\ -\nabla g_1(\bar{x}) \\ -\nabla g_2(\bar{x}) \\ \cdot \\ \cdot \\ \cdot \\ *-\nabla g_n(\bar{x}) \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -\nabla g_{t+1}(\bar{x}) \\ -\nabla g_{t+2}(\bar{x}) \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ -\nabla g_r(\bar{x}) \end{bmatrix}$$

we see that $Az < 0$, $Bz = 0$ has no solution $z \in X^0$.

So, by the Theorem on alternative, Form II, there exists

$(\bar{\mu}_0.\ \bar{\mu}_{I_1}, \bar{\mu}_{I_2} \in R \times R^t \times R^{r-t}$ such that

$\bar{\mu}_0 \nabla f(\bar{x}) + \bar{\mu}_{I_1} \nabla g_{I_1}(\bar{x}) + \bar{\mu}_{I_2} \nabla g_{I_2}(\bar{x}) = (\bar{\mu}_o, \bar{\mu}_{I\,1}) \geq 0.$

We note that $\bar{\mu}_0 \neq 0$, for if $\bar{\mu}_0 = 0$, *then definitely* $\bar{\mu}_{I_1} \neq 0$.

So we have $\bar{\mu}_{I_1} \nabla g_{I_1}(\bar{x}) = 0$ [*since* $\nabla g_{I_2}(\bar{x}) = 0$], *where* $\bar{\mu}_{I_1} \neq 0$.

Also $\nabla g_{I_1}(\bar{x})\, z > 0$ [by construction of I_1] . But this implies that no component of $\nabla g\ I_1(\bar{x})$ is zero. in particular, let

$\nabla g_s(\bar{x}) \neq 0, \quad 1 \leq s \leq t.$

Then, as $\bar{\mu}_s \neq 0$,so we have $\bar{\mu}_s\, g_x(\bar{x}) \neq 0$, which

contradicts the fact that $\bar{\mu}\ I_1 \nabla g_{I1}(\bar{x}) = 0$. Let

$\bar{\mu}_J = 0$ where $J = \{ i : g_i(\bar{x}) > 0 \}$.

So $I \bigcup J = \{1, 2, 3, \ldots, m\}$ and we have

$\bar{\mu}_0 \nabla f(\bar{x}) + (\bar{\mu})_I \nabla g_I(\bar{x}) + \bar{\mu}_J \nabla g_J(\bar{x}) = 0$,where

$\bar{\mu}_0 \neq 0, \ (\bar{\mu}_0, \bar{\mu}_I) \geq 0, \bar{\mu}_J = 0.$

we write $\bar{\lambda}_I = \dfrac{\bar{\mu}_I}{\bar{\mu}_0}$ and $\bar{\lambda}_J = \dfrac{\bar{\mu}_J}{\bar{\mu}_0}$

Thus we have, $\nabla f(\bar{x}) + \bar{\lambda}_I \nabla g_I(\bar{x}) + \bar{\lambda}_J \nabla g_J(\bar{x}) = 0, (\bar{\lambda}_I, \bar{\lambda}_J) \geq 0$

i.e. $\nabla f(\bar{x}) + \bar{\lambda} \nabla g(\bar{x}) = 0$, 0

This completes the proof.

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Photoconductivity of chemically deposited CdS:Al films

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Abstract Trivalent aluminium inereases the photoconductivity of CdS many times. A maximum value of photocurrent to dark current ratio equal to 6278 is found. Resutls of rise- decay studies, excitation and transmission spectra are discussed and values of tap depths are presented.

Key words. Photoconduction : CdS: Al films; chemically deposited films.

1. Introduction

Photoconductivity of CdS films are important due to their wide applications in xerography, photovoltaic solar energy conversion and thin film transistor electronics. Chemical deposition forms one of the simplest techniques and was attempted by a number of workers (Mondal et al 1983: Danaher et al 1985: Kaut et al 1980) though with little success from technological point of view. Recently Nair et al (1984, 1987) through their work on optoelectronic characteristics of solar assisted chemically deposited films have claimed the status of this technique as a viable technology. Further, Bhushan and Sharma (1990 a & b; 1991) have reported many fold improvement in the photoconductvity of chemically deposited rare earth doped CdS films. Like triple valence rare earths other elements can also play a similar role. With this view, trivalent Al was attempted and results of chemically deposited CdS: Al films are reported in the present paper.

2. Experimental

For preparing CdS films, IM solutions of each cadmium acetate and thiourea (both AR Grade) were prepared in distilled water. This concentration was selected on the basis of highest photoresponse. A Complex was prepared by definite volumes of these solutions along with triethanolamine and 30% aqueous ammonia (both AR Grade) such that the pH of the complex was around 11. The depositions were carried out at room temperature by dipping cleaned commercial quality microscopic glass slides of dimensions 24 mmx 75 mm for definite times between2 to 22 hrs. For preparing doped films. definite volume of 0.01 M solution of aluminium chloride (AR Grade) was mixed in the original mixture and then the depositions were made. After depositions, the films were cleaned by flushing the distilled water and then dried. Annealing was done at a temperature of 400^{o}c for different times between 1 to 9 minutes. The films prepared in this way were found to be quite stable.

Coplanar silver paint electrodes of 2 mm width and 24 mm length were applied on the surface of the films at a separation of 1mm. 300 W and 1 KW commercial bulbs were used as excitation sources for rise- decay and spectral (excitation) studies respectively. The photocurrents were recorded by either a nanometer (Scientific Equipments, Roorkee) or a digital multimeter (Keithley 160). For transmission spectra a Varian DMS-100 spectrophotometer was used.

3 Results and Discussions

3.1 Dark current

Fig.1 shows the nature of dark current with applied voltage for different CdS;Al films prepared at different deposition times alongwith those of CdS films for comparison purposes.

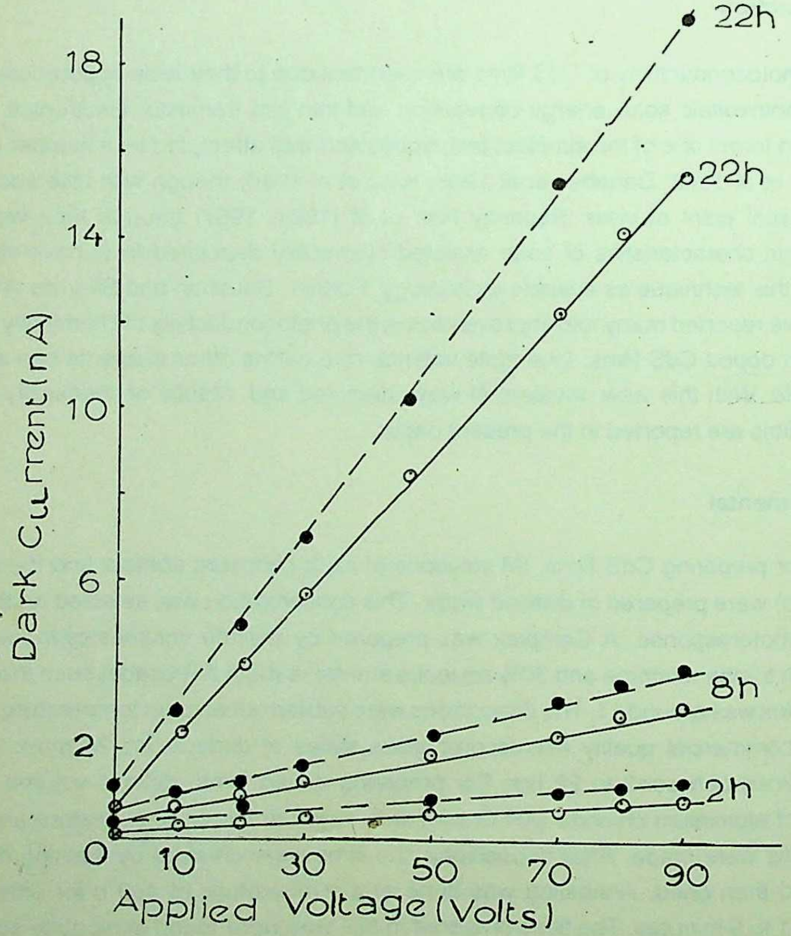

Fig 1 : Deperdence of dark current on applied voltage at different deposition times forundoped CdS (•) and CdS:Al (o)

Since the variation is represented by straight lines in all the csses, it is concluded that the electrodes used are of ohmic type. Further, due to substitution of Al the dark current is reduced in all the cases and the variation becomes faster at higher deposition times.

3.2 Rise and decay curves

Fig.2 shows rise and decay curves for CdS: Al films prepared at different deposition times. From these curves it is inferred that the maximum photoresponse appears for the films prepared at a deposition time of 8h. Reduced photocurrent at a deposition time of 22 h may be due to increase in thickness. For undoped CdS, again the maximum photoresponse is obeserved for 8 h deposition time. This behaviour is also shown in this figure. The rise and decay curves for CdS : Al films prepared at different concentrations of $AlCl_3$ are shown in fig.3 from which it is observed that a concentration of 5 ml of $AlCl_3$ give the maximumphotocurrent.

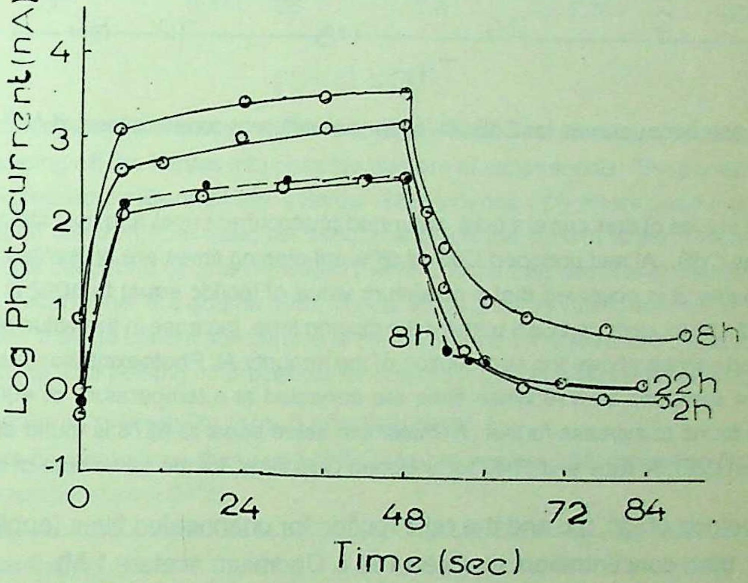

Fig 2. Rise and decay curves for CdS:Al (o) films at different deposition times and for undoped CdS (•) for 8 hrs deposition.

The general nature of rise and decay curves can be described as a combination of fast variation in the beginning followed by a saturation. This behaviour is explained due to generation and recombination effects i.e. when recombination starts dominating, the saturation effect appears. Increase in photocurrent in CdS: Al films may be due to release of electrons from Al under photoexcitation. This is also indicated from fig.3 which shows maximum photocurrnt at a particular concentration of $AlCl_3$. It is also found that the observed decay in the present case are slower compared to those of CdS crystals (Bube 1960). It is suggested that the prolonged decay of the photo induced mobility to dark mobility level may be contributing factor to the prolonged photocurrent (Bhushan & Sharma 1990a & b).

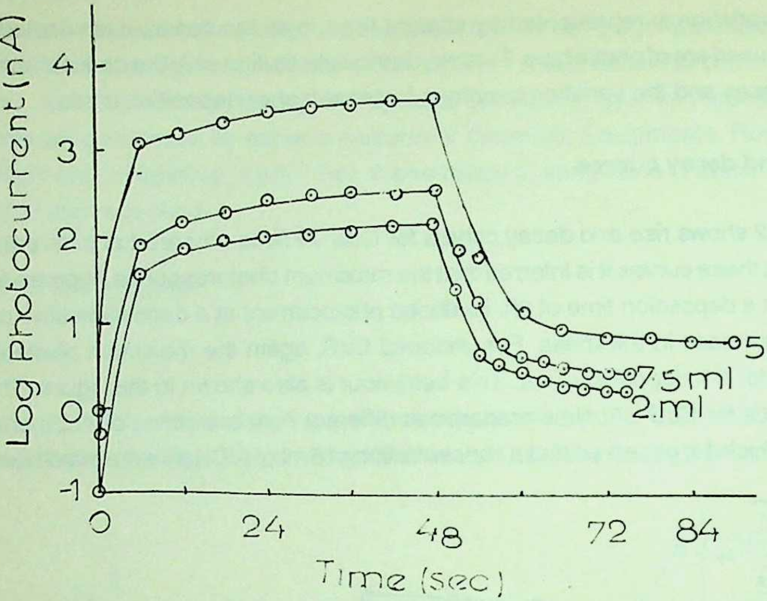

Fig 3. Rise and decay curves for CdS: Al (o) films at different conentrations of $AlCl_3$ for 8 hrs

The values of dark current (Idc), saturated photocurrent (Ipc) and the ratio Ipc/Idc for the systems CdS : Al and undoped CdS at different dipping times are presented in table1. From this table, it is observed that a maximum value of Ipc/Idc equal to 4092 is found for CdS:Al and 651 for undoped CdS both for 8 h dipping time. Increase in the value of this ratio in the doped sample shows the contribution of the impurity Al. Photoexcitation may release the valence electrons from Al.When films are annealed at a temperature of 400°c, these values are found to increase further. A maximum value equal to 6278 is found at 3 minute annealing of CdS: Al films and 1540 for undoped CdS films for the same time of annealing.

Table 1: values of Idc, Ipc and the ratio Ipc/Idc for unannealed films (applied voltage=30V: bath concentration: thiourea 1 M & Cadmium acetate 1 M)

Deposition time (hrs)	Undoped I_{dc} (nA)	CdS I_{pc} (μA)	I_{pc}/I_{dc}	CdS : Al($AlCl_3$=5ml) I_{dc} (nA)	I_{pc} (μA)	I_{pc}/I_{dc}
2	0.4	0.01	25	0.1	0.398	995
8	0.7	0.465	651	1	4.092	4092
22	5.9	0.038	6.44	5.6	1.835	327

This behaviour is quite similar to those observed by Karanjai and Desgupta (1980) for CdS: Cu, Cl films and by Bhushan and Sharma (1990 a & b) for rare earth doped CdS. Bhushan and Sharma (1992) also found that due to annealing a combination of both hexagonal and cubic phases is observed. Other values at different annealing times are presented in table2.

Table 2: Values of Idc, Ipc and the ratio Ipc/Idc for annealed films at 400°c (applied voltage=30V, bath concentraton: thorurea 1 M & cadmium acetate 1 M, desposition time 8 hrs)

Annealing time (min.)	Undoped CdS			CdS : Al($AlCl_3$=5ml)		
	I_{dc} (nA)	I_{pc} (µA)	I_{pc}/I_{dc}	I_{dc} (nA)	I_{pc} (µA)	I_{pc}/I_{dc}
1	0.9	0.776	862	1.9	9.56	5031
3	0.8	1.232	1540	1.8	11.3	6278
5	0.8	0.744	930	1.7	8.75	5147
7	0.7	0.633	905	1.7	8.36	4917
9	0.6	0.441	735	1.6	6.76	4225

The decay curves showing hyperbolic nature have been analysed to calculate trap depths by peeling off the curves into possible number of exponentials. The photocurrent at a time t is expressed by (Buragoham & Barua 1983) $I_t=I_o \exp(-pt)$ where p is the probability of escape of an electron from traps per second and I_o is the current at the moment light is interrupted. p is described by $p=s \exp(-E/kT)$ wher E is the trap depth and S is the number of times per second that the quanta from crystal vibrations (phonons) attempt to eject the electrons from traps to conduction band, k is the Boltzmann constant and T the temperature in absolute units. The peeling off procedure for one of the curves is represented in fig 4. The values of p have been calculated from different straight lines thus obtained. Using (Subasic & Ceric 1972 $S=10^{11}\ S^{-1}$, values of E are calculated. These values at different concentrations of $AlCl_3$ are listed in table 3. Thus, traps with two kinds of E values are found to be effective. However, no regular variation is observed due to change in concentration of $AlCl_3$.

Table 3: Values of probability term p and trap depth E

(Bath concentration : thiourea 1 M & Cadmium acetate 1 M; deposition time 8hrs; room temperature = 34°c)

Concentration ion of $AlCl_3$ (ml)	P1	P2	E1 (eV)	E2 (eV)
2.5	0.6538	0.2179	0.682	0.711
5.0	0.7973	0.1807	0.677	0.776
7.5	0.4917	0.2311	0.689	0.685

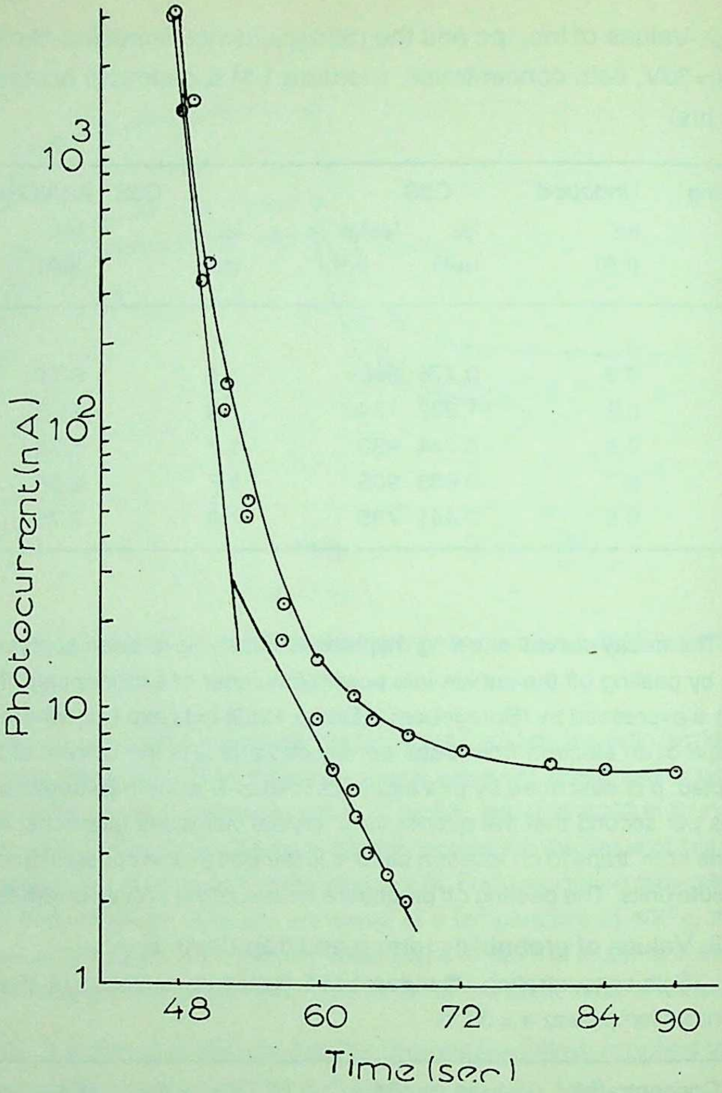

Fig4: Peeling off the decay curves for CdS: Al films ($AlCl_3$-5ml)

3.3 Transmission Spectra

The transmission spectra of different films are presented in fig.5. For CdS, the spectra are presented under annealed and un- annealed conditions.In all the curves a drop at about 510-520 nm is observed which corresponds closely to the band gap of CdS. Sharp fall in curve number 1 indicates crystallne nature of CdS due to annealing. A maximum transmission of about 70% is found in annealed films of 2 hrs deposition. Furhter, increasing deposition time reduces the transmission to some extent. Transmission spectra of unannealed CdS:Al films prepared at two deposition times (2 and 8 hrs)are also shown in this figure. Drop at about 520 nm is again indicative of CdS. The change in the nature of curves compared to those of undoped CdS may be due to presence of impurity Al.

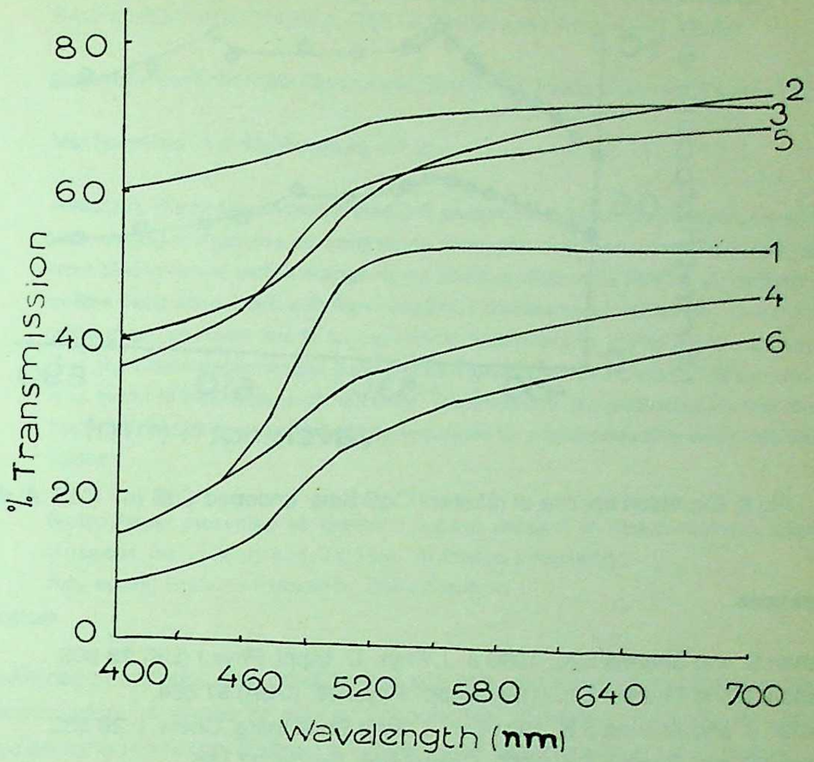

Fig. 5: Transmission spectra of differenct CdS films: 1. annealed undoped CdS (Deposition time 8 hrs), 2. annealed undoped CdS (deposition time 2 hrs). 3 unannealed undoped CdS (Deposition time 2 hrs), 4. unanealed undoped CdS (deposition time 8 hrs), 5. unannealed CdS : Al (deposition time 2 hrs) and 6. unannealed CdS:Al (depositon time 8hrs).

3.4 Excitation spectra

The cxcitation spectra of undoped CdS and CdS:Al films are shown in fig.6. The peak at about 510 nm in both the cases in due to the host CdS. Another peak at about 575 nm may be due to the presence of some impurity. A comparatively strong peak at 620 nm in the spectra of CdS:Al films is indicative of the presence of impurity Al.

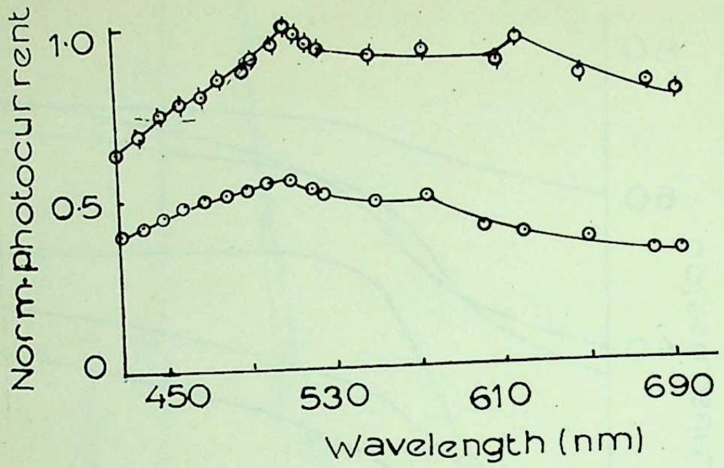

Fig 6: Excitation spectra of different CdS films: undoped CdS (o) CdS :Al Φ

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Photoluminescence of doubly activated ZnS phosphors

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Abstract. The hydrogen coactivated ZnS phosphors with constant copper concentration and varying manganese concentrations show photoluminescence with colour varying from blue-green to yellow orange range under excitation by 3650 Å UV-radiation. The yellow band associated with Mn- transitions dominates at higher Mn- concentrations while green emission due to copper almost disappears for higher Mn- concentration. The photoluminescent output is studied as a function of temperature. The samples were also found to exhibit photoconductivity. The presence of coactivators like chlorine and hydrogen has been observed as pre-requisites for photoconductive and photoiuminescence.

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Key words : Photoluminescence, ZnS phosphors.

1.Introduction

Sensitized phosphors may be classified into two categories: non- photoconductive and photoconductive. A series of nonphotoconductive phosphors were developed and investigated by various workers (Botden & Kroger 1948, Schulman et al 1950, Botden 1950 & Ginther 1954). On the other hand, investigations on photoconductive phosphors are based on sulphide type compounds activated by manganese or samarium as one of the activators. Shionoya (1954 1956) made studies on luminescent and optical properties of self and manganese activated ZnS.

Photoluminescence in ZnS phosphors activated by copper or silver has been a subject of numerous investigators. Comparatively less work has been done on the doubly activated ZnS phosphors. Tolstoi (1961) prepared ZnS phosphor doubly activated by Ag and Co. He observed a flash like rise of luminescence intensity in ZnS : Co and ZnS: Ag, Co phosphors. Levshin et al (1961) prepared ZnS:Ag,Sm phosphor system and observed a single trap level due to Ag and three trap levels due to Sm in ZnS host lattice. Hazimura and Ato (1967) prepared ZnS,Cu /Ag,Cl phosphor and observed blue and green bands when irradiated by different type of radiations, viz UV, X-rays, α-rays. Pandey et al (1971) observed double peak emission under irradiation by UV (3650 A^{o})in case of ZnS:Ag.Mn(Cl) phosphor. They also found dominance of Mn-emission with increasing concentrations of manganese. The photoluminescent studies in (Zn,Cd)S: Ag, Ni, Co were recently made by Mandouh (1987) and in case of ZnS:Mn by Yeong Jan Shin et al (1987). Mandoudh suggested a theoretical model and identified Ni and Co as quencher for photoluminescence. Summarizing, we conclude that presence of transition elements particularly Mn or Co plays an important role in the photoluminescence of doubly activated II-VI compounds.

The ZnS:Cu,Mn phosphor system has been found as an effective triple band emitting electroluminophors (1986), but probably no effort has been made to explore the possibility of photoluminescence in such doubly activated luminophors. Therefore, the present work was undertaken in order to investigate photoluminscence in doubly activated and hydrogen coactivated ZnS:Cu,Mn(H) phosphor system. Mainly the behaviour of Mn-band has been investigated depending upon its concentration and a comparative study is done in the case of ZnS:Mn and ZnS: Ag, Mn.

2 Experimental

The activators of luminescent grade ZnS i.e. Mn, Ag and Cu were taken in the form of their sulphates and acetates. An organic compound fluorene (C_{13} H_{10},m.p. 114°, b.p.295°C) was used to liberate hydrogen as coactivator. The samples were fired in loosely capped silica tube in1-m long tubular furnace (silica made) at about 1000°C for one hour. The system gives good result when firing is done under the flowing atmosphere of H_2S. The luminescent cells were prepared by sandwitching the dielectric suspended powder phosphors between two plane transparent conducting mica sheets. Generally sedimental type cells were prepared and found suitable for these studies. The panel was excited by allowing 3650 A° UV using mercury lamp with appropriate filters. The spectral distribution of luminescent output was measured with the help of a prism monochromator (M/s Scientific and Electronics, New Delhi) coupled with photomultiplier tube (RCA 931A). The output current was recorded with a sensitive nonoammeter (NM-122 ; M/s Scientific Equipments, Roorkee).

3. Results an discussions

A series of doubly activated phosphors with constant copper concentration and varying manganese concentrations show luminescence with colours varying from blue and / or green to yellow orange under 3650 A° excitation. Some typical results are shown in fig.1 The emission of manganese dominates with higher manganese concentration. In the figure brightness intensities are normalized to show the peak shifts on wavelength scale. Because of the overlap of the Cu-band with Mn-band the samples showing both the copper and manganese emissions have broad bands.

The sample containing manganese less then 10^{-4}g/g ZnS, show the yellow emission scarcely; those containing 10^{-3} g/g show distinctly both the copper and manganese emissions and those with manganese 5×10^{-3} g/g or more show predominant Mn-emission while copper emission gradually decreases and ultimately disappears at very high Mn- concentration. This observation confirms that the yellow emission is solely due to manganese impurities (in host). Here energy transfer from Cu to Mn centres may also be considered important for such changes.

The emission peaks of copper and manganese are located at about 4800 A°, 5250 A° and 5750 A° respectively. The intensity of blue peak is very weak under UV-irradiation. The copper peak has a tendency to shift slightly towards longer wavelength with the increase

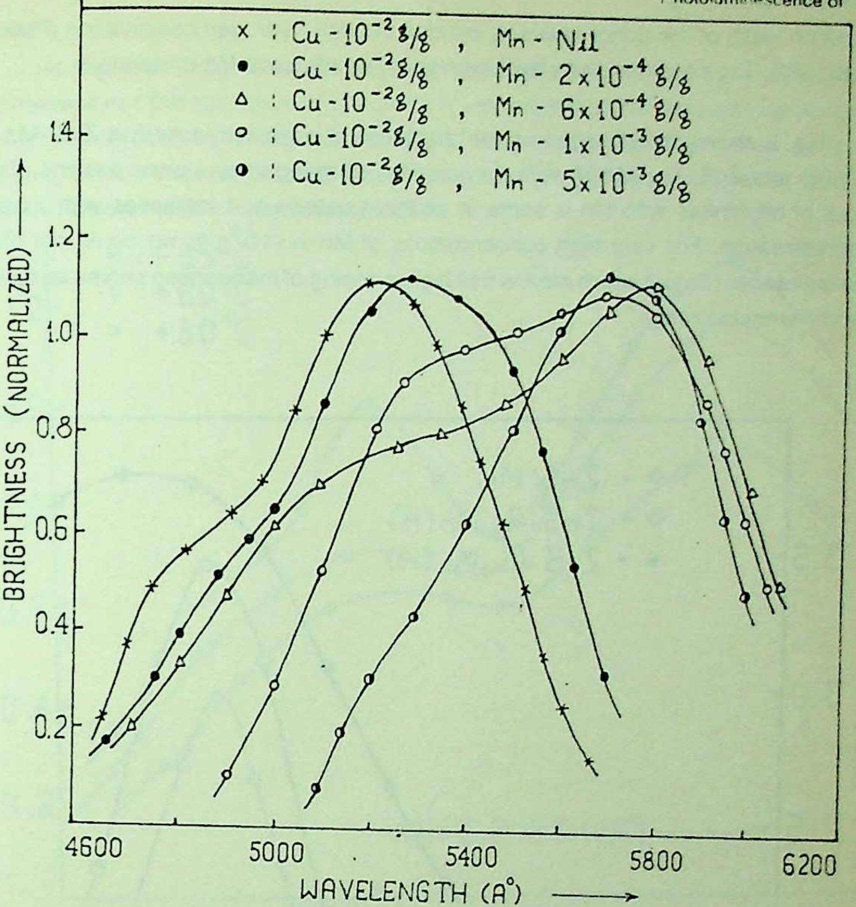

Fig 1 Spectral distribution of photoluminescent brightness of ZnS:Cu,Mn (H) under 3650 A° excitation.

of Mn- concentration. The reason for this effect is considered to be that since Mn- is incorporated substitutionally into ZnS, forming a solid solution and causing the expansion of lattice, resulting a new binary compound like (ZnS+ MnS). Thus, forbidden gap of the host lattice is made smaller and then the energy of copper emission becomes smaller (Pandey et al 1971).

The observed emissions almost resemble to those of Shionoya (1954,1956) in case of ZnS: Cu, Mn (Cl). The yellow band corresponds to the intraionic transitions of manganese incorporated in ZnS and green to the transition from coactivator level near conduction band to Cu-level. The weak emission of blue may be assigned to the direct transition from bottom of conduction band to Cu-level as usual.

The role of hydrogen coactivator is found significant as it enhances the intensity and maintains the simultaneous existence of various emissions with the change of intensity of exciting radiations. This phenomenon of 'no shift' may be explaied as being due to narrower distribution width of the donor levels so created through hydrogen coactivation (Prakash & Mohan1969). The coactivation by hydrogen does not influence Mn-emission.

Fig. 2 shows the change in total brightness of emission incase of ZnS: Mn, ZnS: Ag.Mn(H) and ZnS: Cu, Mn(H) with the variation of manganese concentrations. The behaviour of brightness with Mn is same in all three cases i.e. it increases with increasing Mn-concentration. For very high concentrations of Mn (>>10^{-2}g/g) we could not observe photoluminescent output which means that higher doping of manganese serves as quencher for whole emission range.

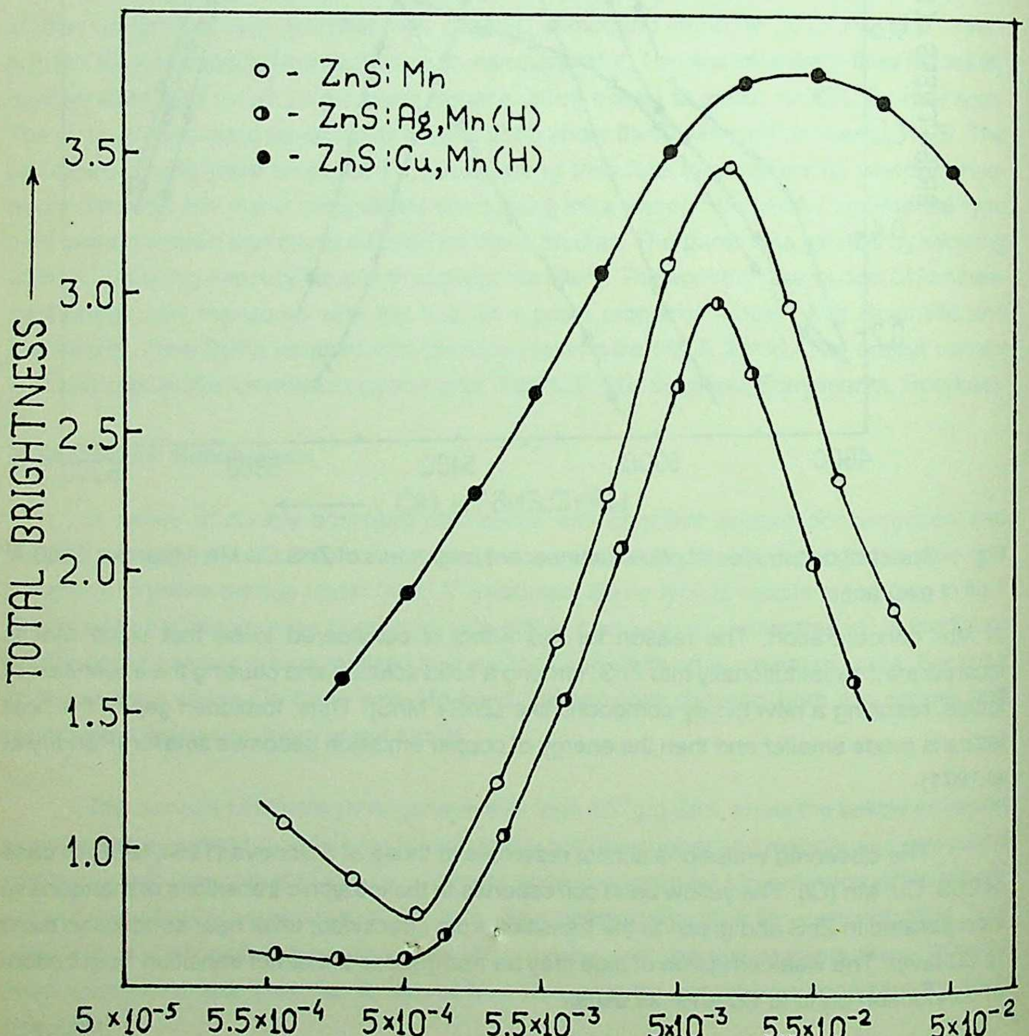

Fig. 2 Total brightness of Mn-yellow band vs concentration of Mn-activator

As depicted in fig. 3 we observed the spectral distribution of the photoluminescent brightness under the excitation of UV (3650 A°) in respect of the temperature dependence. The photoluminescent intensity decreases with temperature inall three cases. For the same change of temperature, brightness intensity of green and yellow emissions fall rapidly while in case of blue emission it is relatively lower.

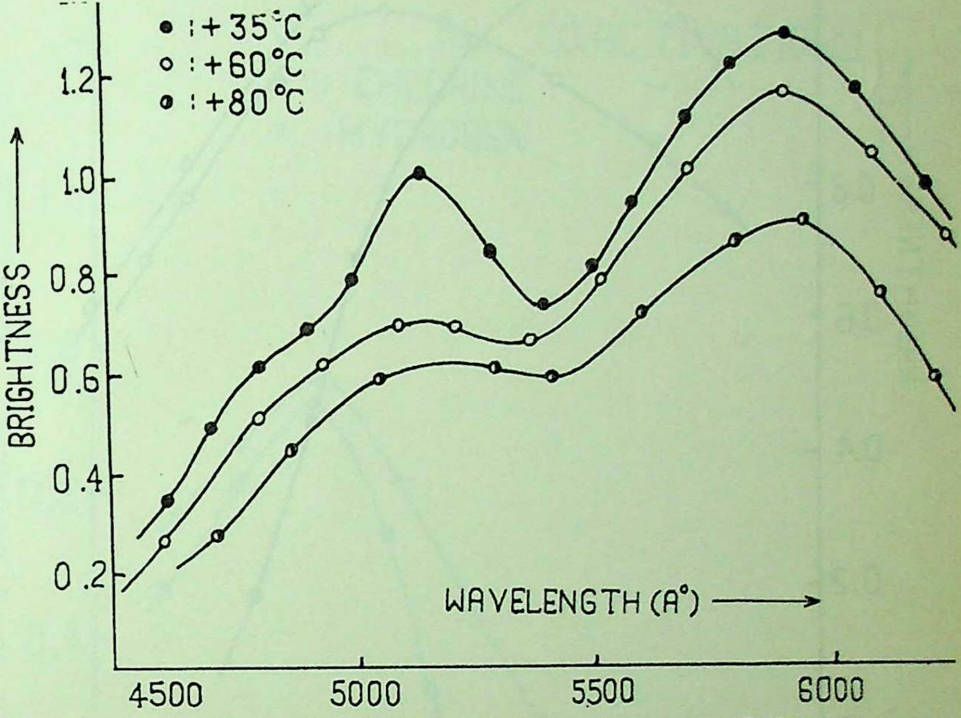

Fig. 3: Curves showing spectral distribution of photoluminescence of ZnS:Cu, Mn(H) triple band emitting sample at different temperatures.

Blue emission dominates as seen in fig. 4, at lower temperatures. As is clear from the figure, the yellow emission curve crosses the blue and at further more lower temperatures (near-30° C) intensity starts decreasing. One possible reason may be that yellow centres transfer energy to blue centres contributing to the rise of blue emission Similar type of energy transfer may also take place from green centres to blue centres at some what higher temperatures.

We have also studied photocurrent response for the given sample. A Slit type of sedimental cell was used for this purpose, on a conducting mica sheet keeping a small nonconducting gap as slit. This gap on the surface of mica sheet was nonconducting and the remaining portion of the surface was conducting due to thin film deposition of SnO_2 on the same surface. The fine powder of phosphors was poured in the gap with few drops of alcohol which is quite nonreactive. Thus a uniform layer of phosphor was deposited while alcohal

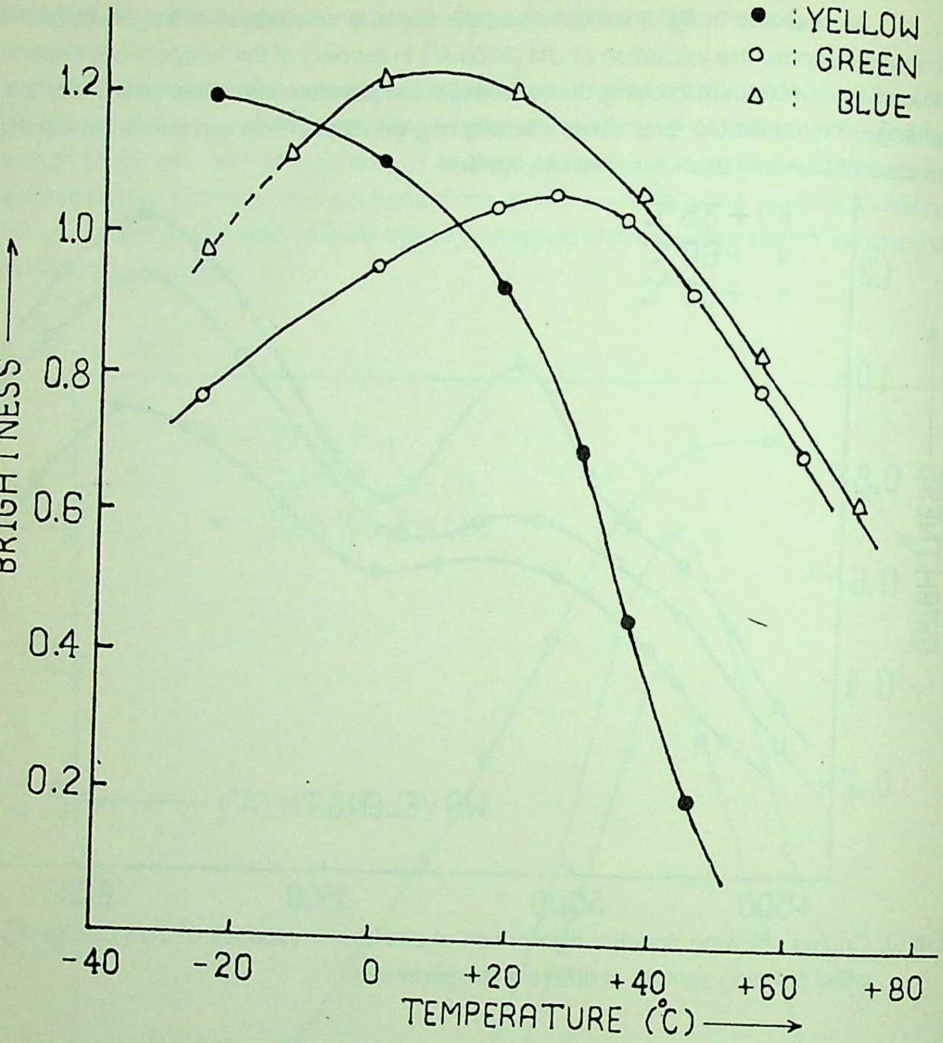

Fig. 4: Photoluminescent brightness with temperature under photoexcitation by 3650A° UV-radiation for isolated emission band of ZnS:Cu,Mn(H).

was dried out. The output photocurrent is very small under 3650 A° excitation. Photocurrent was recorded by means of a sensitive nonoammeter.

In fig. 5 the relative photocurrent response of a series of ZnS Cu, Mn with a constant Cu concentration and varying manganese concentrations are given as a function of manganese concentration. It is shown that the photo- conductive response changes abruptly to the smaller value at the manganese concentration at which the manganese emission becomes distinct. As is clear from figure, chiorine or hydrogen plays a significant role as coactivators for photoconductivity.

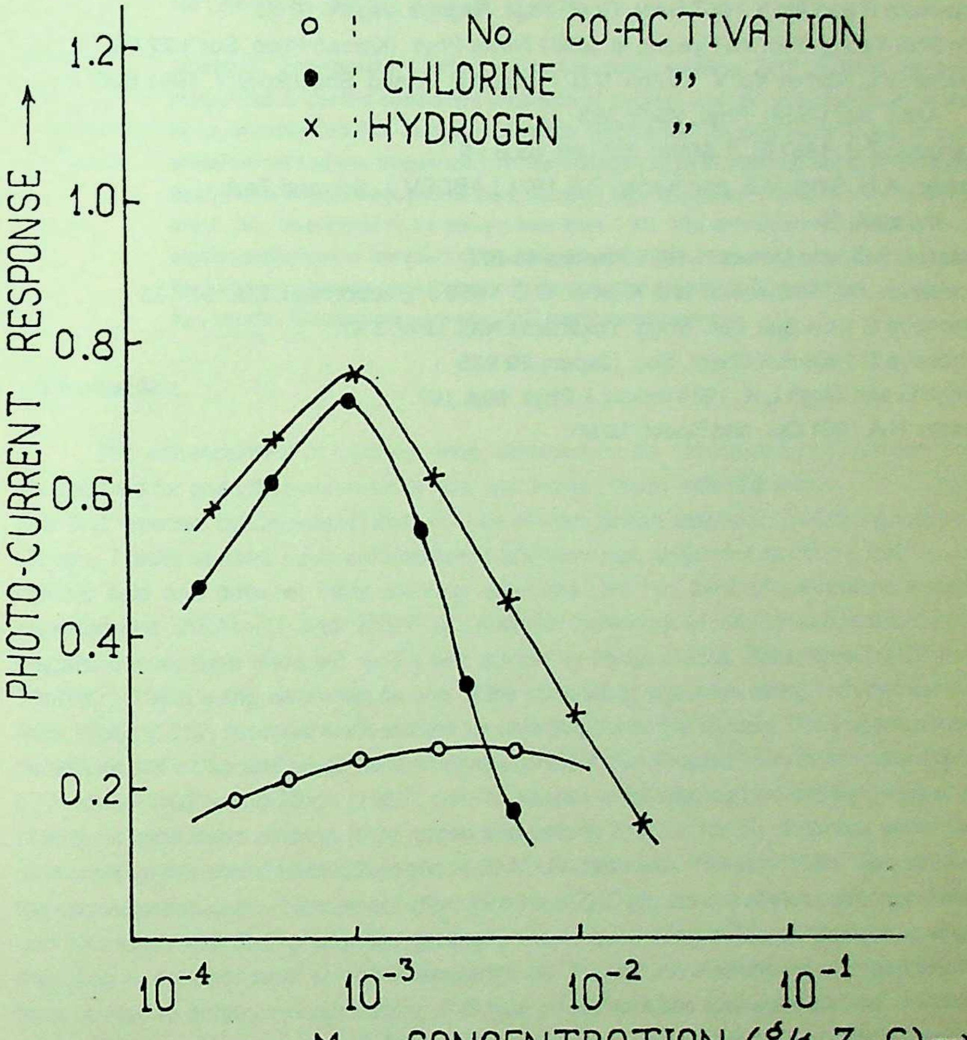

Fig. 5: The relative pholocurrent response of a series of ZnS:Cu, Mn as a function of Mn-concentration.

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Radiation controlled Electroluminescence in double band emitting ZnO-base electroluminophors

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Abstract. Enhancement effect in double band emitting ZnO:Cu,M(H) electroluminophors, under the concurrent excitation of a.c.field and UV- radiation (3650 A°) has been investigated. The emission in green and yellow regions were found to show enhancement at low frequencies (in the voltage range of 200-700 volts) of excitation field. While at high frequencies the quenching was observed. The maximum enhancement ratio was found to be always less than 1.33. The enhancement effect is found significant for yellow emission of electroluminophor with increasing excitation voltage of field and manganese concentration in presence of constant UV- radiation.

Key words ; Electroluminescence, ZnO electroluminophors

1. Introduction

The enhancement of luminescence, obtained by the combination of radiation and electric field for phosphor materials is now well known. X-ray induced enhancement effect was first reported by Destriau (1954) in case of manganese activated Zn-Cd)S phosphor. Cusano (1955) studied such enhancement phenomenon under the concurrent action of electric field and different other exciting radiations for thin films of electroluminescent materials like ZnS:Mn(Cl) and ZnS:P As. A similar behaviour of polycrystalline phosphors dispersed in suitable dielectric media was strdied by Patek (1959), Ballentyne (1961) and Thornton (1962) using ultraviolet as one of the stimulating agencies along with the electric field. Wood (1957) reported such studies for unactivated single crystals. The instantaneous behaviour of the blue and green band emitting ZnS electroluminophors has been investigated by Prakash (1971) and Singh (1967). Similar studies were also carried out by Singh et al (1992) for triple band emitting (blue, green and yellow) ZnS:Cu,Mn(H) phosphor under the joint action of sinusoidal electric field and 3650 A° UV- radiation. Prakash (1983) also studied the ultraviolet induced enhancement effect for a set of ZnO phosphors at various tempeatures and concluded that ZnO phosphors generally shows quenching at low temperatures while they lead to enhancement at higher temperatures. Though an abundance of experimental facts related to enhancement study of ZnS type phosphors are available, but the radiation controlled electroluminescence in ZnO-base phosphors still needs rigorous characterization.

The common findings of all the workers show that when a phosphor layer is made luminesce under the concurrent action of two different stimuli, the net output of light is generally found to be non-additive. The enhancement ratio 'R' has been defined by (Ivey 1963)-

$$R = \frac{B_{E+R}}{B_E + B_R} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$R<1$ for quenching and $R>1$ for enhancement

where, B_{E+R}=Brightness under the joint action of electric field and UV-radiation

B_E=Electroluminescent brightness

B_R=Luminescent brghtness due to UV-radiation

The enhancement ratio 'R' was always observed to be less than 1.33.

The present contribution reports the experimental results of radiation enhancement of double band (green and yellow) emitting ZnO:Cu, Mn (H) electroluminophor.In this sequence, both static and time dependent behaviour of electroluminescent emission have been studied under an a.c. electric field with a concurrent excitation by 3650 A° ultraviolet source.

2. Experimental

The electroluminescent cells of usual paralled plate type was excited by a.c. field at different voltages and frequencies with the help of audio-frequency oscillator/wide band amplifier unit. Ultraviolet radiations (λ=3650 A°) were obtained from a high pressure quartz mercury lamp after passing its light through the appropriate neutral density filter (from M/S Ealingelectro- optics, Wastford England). Appropriate filters were also used to isolate the electroluminescent peaks of different colour emissions from the phosphors. The photo-current corresponding to the electroluminescent output and also after being modified by ultraviolet irradiation was developed by an RCA-931A (spectral response:3000 A°- 6500 A°) photomultiplier tube, operated with high voltage stabilized power supply (Model EHT-10, from M/S Scientific Equipments, Roorkee) providing regulated anode voltage (1250 vols. d.c. for RCA-931 A PMT).The signals so obtained were fed to an automatic graphic recorder (speed=40 mm/sec.) for recording instantaneous behaviour and otherwise the EL-intensities in terms of currents were measured by a sensitive nanoammeter.The experimental arrangement is schematicaly shown in fig. 1.

3. Results and discussions

The spectral analysis of electroluminescence for ZnO:Cu, Mn (H) phosphor shows two distinct emission bands at about 5250 A° and 5850 A° respectively (curve-1in figure-2).The peak intensitiesa were found to vary with the variation of frequency but no significant shift in their positions on the wavelength scale was observed.Thus the same phosphor when excited by 3650 A° UV-radiation,the respective green peak was observed to be at about 5200 A°,while the yellow peak depicted only a negligible shift towards lower wavelength side (curve-2 in fig. 2).

Fig. 3 shows the variation of enhancement ratio 'R' with the spectral range. With the increase of excitation voltage, a remarkable enhancement corresponding to yellow emission is observed at the cost of quenching of green emission.This suggests the probability that

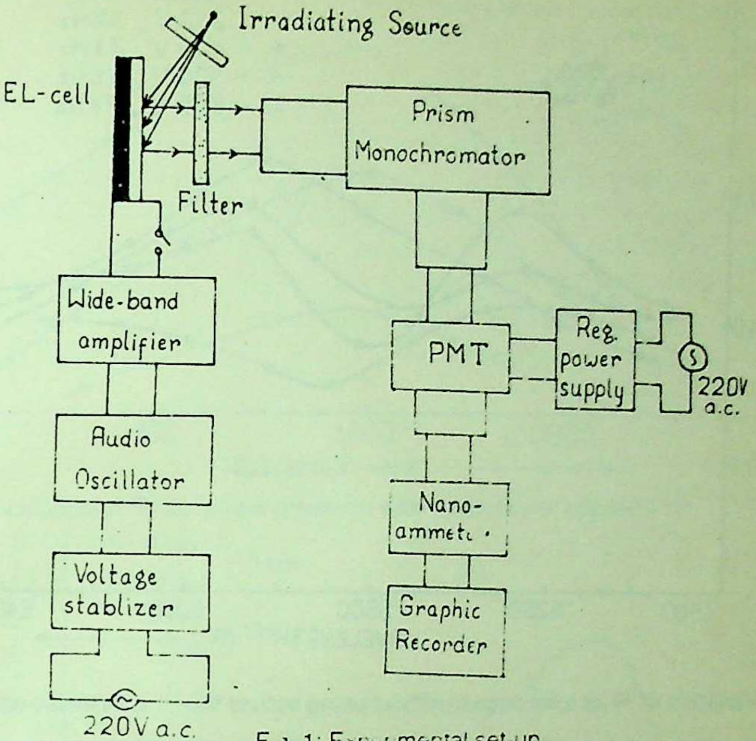
Irradiating Source
EL-cell
Filter
Prism Monochromator
Wide-band amplifier
Audio Oscillator
Voltage stablizer
220V a.c.
PMT
Reg. power supply
220V a.c.
Nano-ammeter
Graphic Recorder

Fig. 1: Experimental set-up

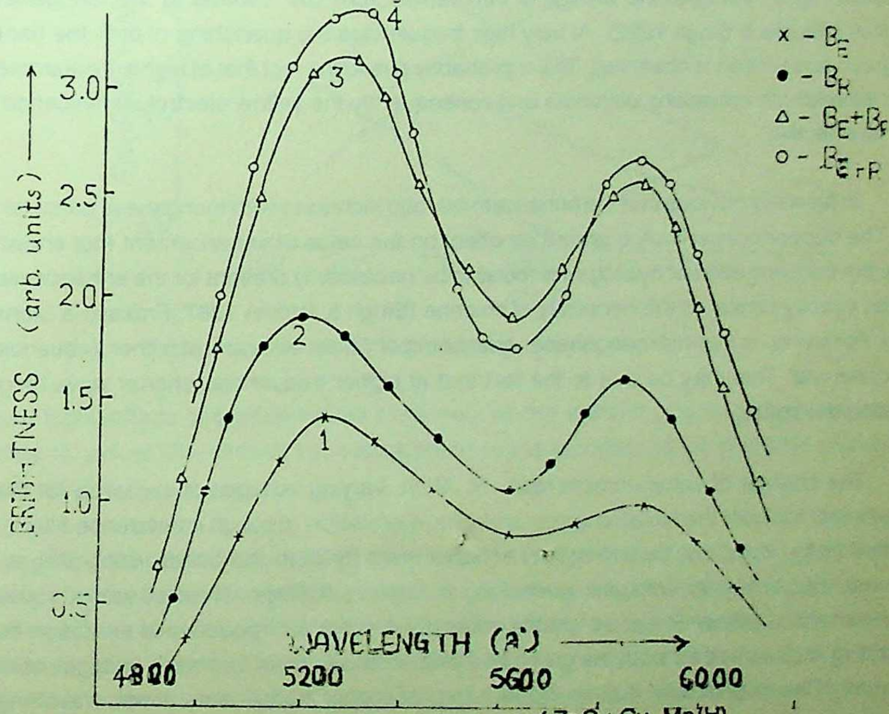

Fig. 2: Spectral curves of ZnO : Cu, Mn(H).

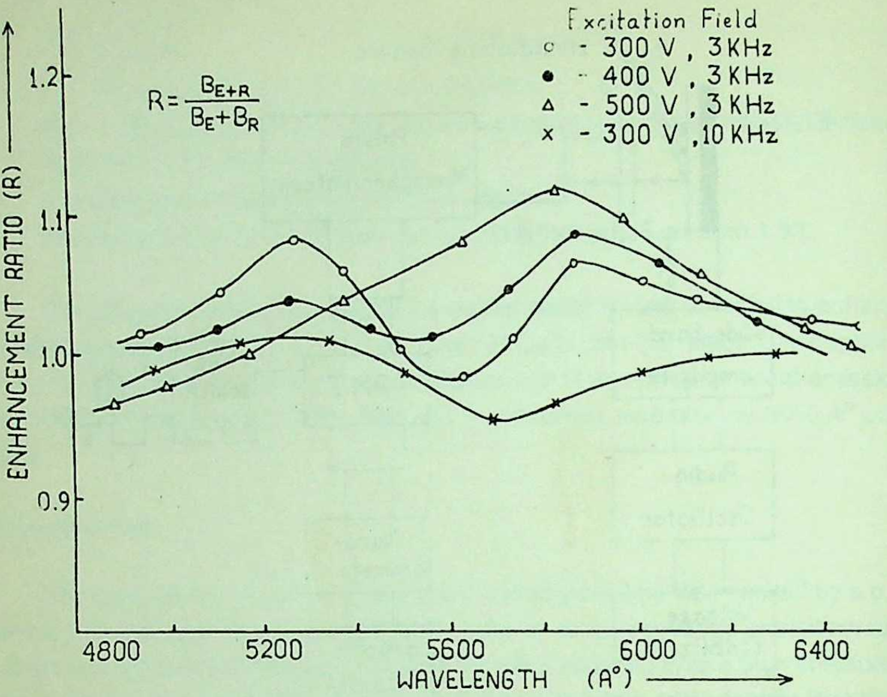

Fig. 3: Variation of 'R' of total output (without using optical filters) with emission wavelength

perhapsat higher voltages,the energy is transferred from Cu- centres to the manganese emission (Shukla & Singh 1992). At very high frequencies the quenching of both the bands (i.e. green and yellow) is observed. This is probabily due to the fact that at higher frequencies, lower wavelength emissions dominate and consequently the yellow electroluminescence is not that effective.

In fig. 4 it is obvious that the enhancement ratio increases with manganese concentration. The copper concentration shows no effect on the value of enhancement (not shown). While the concentration of hydrogen is found to be necessarily present for the enhancement effects, exactly similar to the necessity of chlorine (Singh & Mohan 1967, Prakash & Mohan 1971). For every amount of manganese, quenching of yellow emission at higher frequencies was observed. This may be due to the fact that at higher frequencies shorter wave length emission dominate.

The change of enhancement ratio ' R ' With varying voltages of excitation (at fixed frequencies) for both the isolated green and yellow emission (through interference filters) is depicted in fig. 5. At low frequency. an enhancement for both the bands with voltages is observed. But at higher voltages, quenching is observed for green band with a marked enhancement of yellow emission. On the other hand at higher frequency of excitation field quenching is observed for both the green and yellow emissions at excitation voltages above 350 volts. This is probably due to the fact that at higher frequencies, lower wavelength emission dominates and yellow electroluminescence is not as effective; while at low frequen-

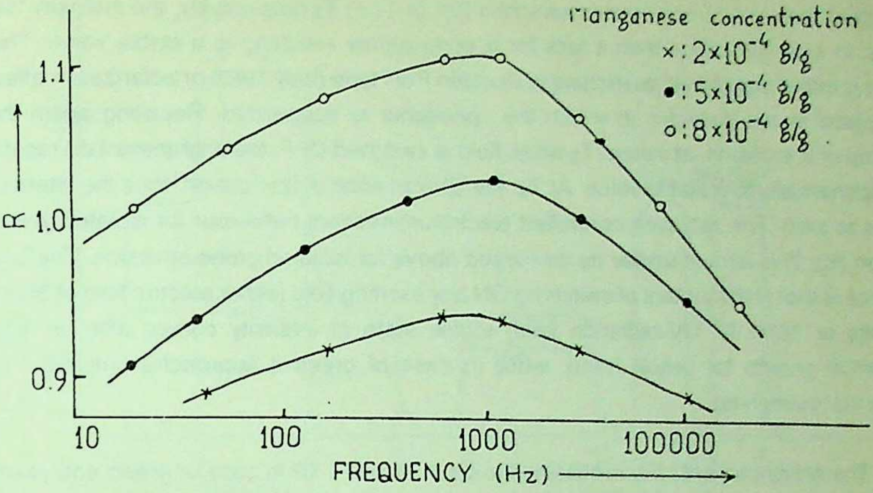

Fig 4 Variation of 'R' for yellow emission with frequency of applied field

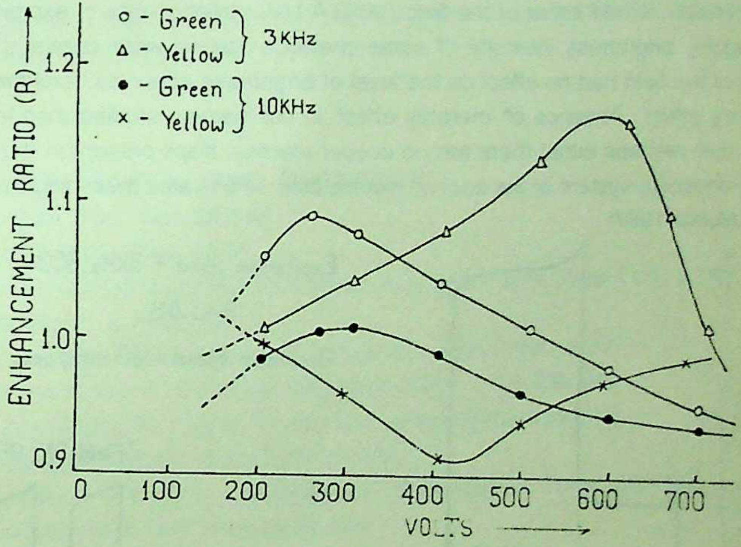

Fig. 5: Variation of 'R' with varying voltages of excitation.

cy and high voltage of excitation field there may be the transfer of energy from green Cu- centres to yellow Mn- centres i.e. sensitization effect becomes more probable (Shukla & Singh 1992).

The time dependent instantaneous nature of enhancement has been recorded on running chart of automatic graphic recorder, for isolated green and yellow emission peaks (fig. 6 and 7). As shown in fig. 6, at instant T1, when electrac field (3 KHz, 300 volts) is switched ON, initially the intensity rises to a certain value then approaches to a stable level. At instant T_2, when UV- radiation of 3650 A^{o} is applied on the previously field excited luminophor, the brightness increases abruptly, then attains a stable value. It is worth mentioning here that when either field or UV- radiation is switched ON at T_1 or T_2 respectively, the intensity first goes up to a certain value then it falls for a while before reaching to a stable value. This perhaps is either due to mild quenching of Gudden Pohl type (Ivey 1963) or polarization effect taking place in the dielectric in which the phosphor is suspended. Recalling again the sequence of discussion, at instant T_3 when field is switched OFF, the brightness falls rapidly then exponentially to a stable value. At T_4, the UV-radiation is also cut-off, thus the intensity reduces to zero. The radiation controlled electroluminescent behaviour for isolated yellow emission (fig. 7) is almost similar as discussed above for isolated green emission. The only difference is that at the instant of switching ON any exciting field (either electric field of 3KHz, 300 volts or 3650 A° UV-radiation field) stable state of intensity comes after a slight exponential growth for yellow band, while in case of green it approaches after a mild exponential quenching.

The enhancement ratio is obtained to be 1.08 and 1.02 in case of green and yellow electroluminescence respectively, confirming established theoretical approaches of Thornton (1962). When either of the field (3650 A UV- radiation or a.c excitation field) is applied again, brightness intensity of same previous values were obtained. The rapid switching of the field had no effect on the level of brightness achieved, eachtime; showing 'no memory effect'. Absence of 'memory effect' in our sample studied also leads one to conclude that perhaps either there are no deeper electron traps present in the considered electroluminescent system or the applied electric field deactivates these traps all together (Singh & Mohan 1967)

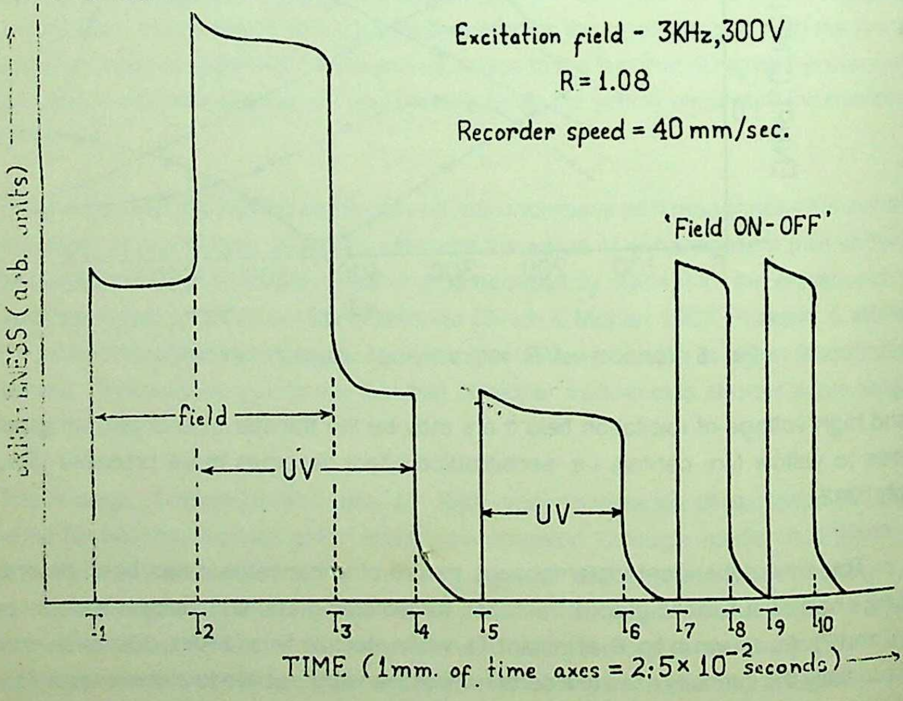

Fig 6 Recorder trace of enhancement for isolated green emission of ZnO:Cu, Mn(H) under concurrent UV irradiation and electric field.

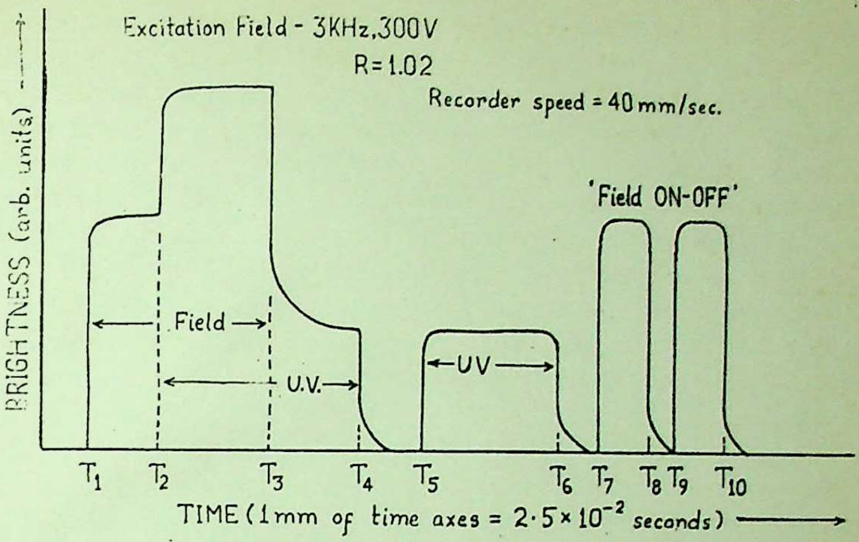

Fig. 7: Recorder trace of enhancement for isolated yellow emission of ZnO.Cu. Mn(H) under concurrent UV irradiation and electric field.

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